MIRI HILAI

Gordon Academic College of Education
Haifa, Israel

PROJECT BASED LEARNING (PBL)
AS A PROMISING CHALLENGE
OF TEACHING MATHEMATICS


Mathematics has always presented a challenge, both for teachers and for pupils, all around the world. Teachers of mathematics of all time periods are interested in having their pupils master the mathematical skills and love math. They deliberate on ways of teaching-learning, because of the tremendous gaps in their pupils’ cognitive abilities and their non-uniform abilities to pay attention and to concentrate. It appears that the main solution in the frontal mathematics lessons is offered to the average pupils, but the main goal is to provide a solution for the entire classroom population.

Over the years I have searched for different ways beyond frontal and individualized teaching, so that I could provide a solution for populations with different needs in the mathematics lessons. My search for alternative ways derived also from the need to promote the achievements and to boost the motivation, interest, curiosity, and enjoyment in the learning of mathematics.

Contemporary research indicates that there is practical innovative learning which is active and involving; it is called project-based learning (PBL). PBL provides a solution for the improvement of the performances in mathematics, for the motivation of the pupils, and for the inspiration of interest and curiosity in and enjoyment from this field of knowledge.

From my experience as a teacher in the past and from the reports of my students in the Gordon Academic College for Education in the PBL course, in such teaching a solution is provided for the different populations in the class. The pupils are engaged in learning in practical and realistic projects that are relevant to their lives. They are more active and autonomous, work cooperatively, and develop patterns of behaviour of independence in learning, self-orientation, and self-regulation.

These skills and patterns of behaviour are important to their lives as adults and cultivate the six functions of the learner that are derived from the curriculum in Israel: sensory-motor, self-direction in learning and in its management, intrapersonal and interpersonal, cognitive and meta-cognitive.

Key words: Project Based Learning, PBL, teaching mathematics
The Search for Alternative Ways in the Teaching of Mathematics

Mathematical thinking has two attributes. The first is its layer nature, which means that mathematics, more than other areas, is built step by step. A mathematical argument is usually long and is based on the stages that preceded it and previous knowledge. The uniqueness of mathematics is its stable and strict connection between the parts of the argument and hence the commitment to accuracy, which is the second trait of mathematical thinking. To meet the complex demands and accuracy of mathematical thinking, strict thinking discipline is required. This discipline means that there is something more important than the researcher or the pupil and his desires. Thus, it teaches an important lesson, which pertains to the status of the individual person and the person in general in the world. Another issue that arises from the requirements of mathematical thinking pertains to the need for the investment of considerable effort, which bears fruit, and for the existence of a place completely different from the starting point.1

Mathematics has always constituted a challenge, both for teachers and for pupils, all around the world. Teachers of mathematics of all times deliberate about ways of instruction, the use of means of illustration, the nature of the assigned tasks, the heterogeneity of the class or the group, and the use of independent or collaborative work. These deliberations derive from their desire to cause their pupils to know and to love mathematics.

It appears that the main solution in the mathematics lessons is primarily for the average pupils. Mathematics teachers, even the most talented of them, cannot plan frontal lesson plans that provide an answer for all the pupils, including pupils with difficulties and gifted pupils. When the inculcation of a new mathematics topic is held in about twenty minutes, the pupils with difficulties will not necessarily understand immediately and will need extended work. The excelling pupils and the gifted pupils understand within the first few minutes and become bored in the following minutes. Assuming that a number of minutes remain till the end of the lesson and a challenge problem is set for the talented pupils, there may not necessarily be enough time to engage in the different ways of solving it.

The advanced technology and the available and frequent stimuli influence our children. The teachers who are at the front of the class appear in the children’s eyes as boring and monotonous figures, who do not move but

---

only change about every hour and a half, unlike the figures who change in the advertisements in the amazing time of fractions of seconds. The ability to pay attention is not equal among all the pupils and when the teacher teaches frontally, certain pupils are not focused on parts of the lesson, or even all of it, and therefore do not understand what is learned, and not necessarily because of learning disabilities.

The advanced technology and the gaps of attention deficit disorder disrupt the normal course of the lesson. Pupils with attention deficit disorder (ADD) stare and lose interest since they missed information that constitutes a basis for full understanding. Pupils with attention deficit hyperactivity disorder (ADHD) disturb the class because of their boredom, which is caused by lack of understanding. The teachers of the 21st century have to be different from the teachers of the past. They have more technological abilities, unlike their predecessors, but very often, they do not try to teach differently. They teach in the same traditional ways while complaining about their pupils’ disorders.

Over the years I have searched for different ways beyond frontal and individualized teaching, so that I could provide a solution for populations with different needs in the mathematics lessons – pupils with difficulties, average pupils, excellent pupils, gifted pupils, and pupils with ADD and ADHD.

My search for alternative ways of math learning derived also from the need to promote the achievements and to boost the motivation, interest, curiosity, and enjoyment in the learning of mathematics. It is clear to every person who works in the field of education that it is important to give to each one of the pupils in the class an extensive, rich, challenging, and supporting solution in the different areas and in the different aspects – intellectual (cognitive), emotional (affective), social and value-oriented, sensory-motor, and spatial aspects. The thinking about and search for ways to achieve this solution continue all the time². My students who are earmarked to be teachers in a few years are more talented than me on technology means. They teach math with IPads and smartphones through Widgets. All the children seem to be active, and there are many smiles around. This is one example of an alternative way. Project based learning (PBL) is one of the alternative ways for meaningful learning and for the development of the learner, training him for the skills of the 21st century. As a teacher, after I discovered the attributes for this method of teaching, my role is to pass it on to my students, student teachers, for its continuation in the educational system.

What is PBL?

Current research studies indicate that there are innovative practical ways of teaching that improve the performances in mathematics and promote the pupil’s enjoyment, interest, and curiosity in this field. From the 1990s there have been three main approaches to active learning – project-based learning, problem-based learning, and inquiry learning. There is considerable similarity among the three approaches, when they all are adjusted today to technology-rich learning environments and considerable information is brought up by the pupils through the Internet.³

In project-based learning, the learner experiences the professional world of the knowledge field, while understanding the interdisciplinary contexts in the knowledge clusters. The project is based on a complex task, derives from a question or challenging problem, aims at a constructive inquiry process and comprehension-based performances. In addition, in work on the project new understandings and skills must be created in the learner.

In contrast to traditional teaching, the learners receive more autonomy in the choice and planning of the time and the steps and more personal and group responsibility in the work process. The projects are connected to and belong to the ‘real’ world, the world outside of the school. They are practical, realistic, and relevant. Sometimes the solutions obtained in the project have the potential to be applied⁴. An example of such a project for which I was a partner in the preparation was a local newspaper prepared by eighth grade pupils in the Reali School in Haifa and handed out to residents of Haifa in the Merkaz HaCarmel neighborhood.

Professor Breithouse⁵, the former head consultant of the chain of schools in London that participated in the writing of a guide through project-based learning, said that:

There will always be adventurers in the field of education who want to break the boundaries of possibilities and are motivated by an enthusiastic belief in what teaching in the school needs to be and can be. This guide can help and provide inspiration to each one to attempt to do as they do and to pave ways for the future for youths, since otherwise during their lives only a small part of the potential innate in them.

The William Smith High School in Colorado is an example of a public high school that was failing and became a flourishing institution following

---

³ Ibidem.
⁴ Ibidem.
the determination of the staff and its commitment to project-based learning. The school pupils were involved in extensive longitudinal research studies of different topics in lengthy periods of time. The engagement in projects encouraged in them critical thinking and creativity and was based on significant content and high quality work. The school staff chose a staff of outside observers that included the parents of school pupils, local business leaders, and educators from other schools. These examiners knew the school background and its vision, observed the teaching, interviewed pupils and teachers, and provided feedback to the leadership team. The initial feedback indicated that the teaching in the classes was not necessarily planned for critical thinking of pupils or cooperation through the networks. The management staff decided immediately upon a redesign, including painstaking teaching, cooperation among the teachers while giving freedom and autonomy in the choice of the topics. They defined a clear goal, a rationale for this goal, steps that should be taken, and methods for the measurement of success in meeting the goal. They recognized that change requires time and resources and the results were not late in coming. The test scores exceeded the average scores of the district and the outside examiners expressed satisfaction with the quality of the teaching and with the significant contents.

Yahel, a homeroom teacher in Israel who teaches using projects, hopes that this teaching method will in the future take the place of some of the units of the high school matriculation examination in which pupils are required primarily to memorize piles of information instead of investigating and discovering new information. In her opinion, the pupils benefit from the process since they are involved in the learning and investigating processes through the presentation of the products and with emphasis on teamwork. “Learning obligates going from passion and from connection to the pupil's everyday life, which causes him curiosity and desire to investigate.”

The populations with different needs in mathematics who learn together in one class sometimes can reach forty pupils. The difference is primarily characterized by the levels of mastery of mathematics and the type of disorder: those with difficulties, those who are of average level, those who are gifted, those who have learning disabilities and attention disorders, those who daydream, and those who are hyperactive. This heterogeneity does not allow an appropriate solution in mathematics for all the populations of the class.

---

It appears that project-based learning constituted an appropriate solution for the needs of the hour in the teaching of mathematics in the past decades in different places in the world. This type of learning caused pupils curiosity and enjoyment. They learned to work in a team and were involved in the processes of the learning, inquiry, and presentation of products. They improved the learning skills and acquired considerable knowledge that is practical, realistic, and relevant to their lives. Their achievements in mathematics improved and the attitude to the study of mathematics improved wonderfully.

Development of competencies Required in the 21st Century

“Analysis of reality shows that components of emotions, cognition, motivation, and behavior are involved in one another and it is difficult to isolate them … the elements penetrate into one another, activate one another, and even increase one another.”

The work program of the Ministry of Education in Israel for the year 2016 focuses on six areas of the learner’s functioning in the 21st century both as contributing to significant learning and as a product of significant learning. As the head of a division, homeroom teacher, and subject teacher of mathematics till recently and as a trainer of student teachers in the present, I do not stop deliberating regarding the ways of teaching that cultivate the six learner functions: cognitive, meta-cognitive, self-direction in learning and its management, intrapersonal, interpersonal, and sensory motor.

The separation into the different fields is artificial and is undertaken for the purpose of the focus and increased depth every time in a certain aspect, and therefore the directive is to look holistically at the learner as a whole person. The idea is to combine between the approaches and the tendencies of the child and the knowledge and skills and practices, when the expected product appears as the individual’s independent behavior in actuality in different life situations. The work plan of the Ministry of Education describes what every function includes:

---

11 Ibidem.
- Cognitive: The ability of the learner to perform a variety of learning and thinking functions, from the level of identification and memory to the levels of application, analysis, synthesis, and assessment.
- Meta-Cognitive: The ability of the learner to think about his processes of thinking and to navigate his learning through thinking about the meaning of the knowledge, the processes, and the strategies that the learner uses.
- Interpersonal: The ability of the learner to manage interpersonal communication that respects the other person and addresses him with a sensitive, tolerant, and considerate attitude, alongside teamwork that takes the other person into account and commitment to the advancement of objectives and shared results.
- Intrapersonal: The ability of the learner to know himself, to feel a sense of self-efficacy, to regulate behaviors and emotions, and to act out of high awareness of the strong and weak points in his functioning.
- Self-Direction in Learning and Its Management: The ability of the learner to choose the goals of his learning, to define them, to plan the learning process while establishing it on the processes of thinking alongside internal motivation and adjustment of the strategies to the goals of the learning.
- Sensory Motor: The ability of the learner to manage a healthy lifestyle, to use body language in an intelligent and conscious manner so as to express ideas and emotions, to realize creativity, and to provide an expression to the modes of learning in a multisensory and motor manner.

Drivers of change are what enable and invite adjustment and flexibility in six main areas: (1) organization of the contents in the curriculum in different structures, (2) organization of time, (3) diversification in the teaching-learning methods, (4) diversification in the assessment methods, (5) organization of the learning spaces, and (6) organization of the learners. As there is the optimal adjustment between the spaces and the processes of teaching-learning-assessment and the learner’s functions, the improvement in the different functions will be more apparent.\(^\text{12}\)

**Independent Educational Authority – Freedom in the Framework**

In the past, educators were assigned to transfer knowledge and be authoritative, namely, to teach and to uphold normal behavior of their students in the lesson. Today the role assigned to the educator has vastly expanded.

\(^{12}\) Ibidem.
The teacher or the homeroom teacher must inculcate knowledge and also develop in the learner functions and skills, such as implementation, analysis, synthesis, assessment, reflective thinking, problem solving, collaboration, persuasion, standing in front of an audience, self-management, self-regulation, interpersonal communication, intrapersonal observation, self-orientation, and sensory-motor skills.

Goskov\textsuperscript{13} maintained that the educator must gradually change the power of the educational authority that acts on him, its direction, and its origin. He must assign this authority to very young educators and educate them to adopt for themselves an educational personality that is appropriate in their opinion, to the point that they will serve as an educating authority for themselves. Every student, with the guidance of an appropriate teacher, can perform in-depth investigation of his character, which will promote him and increase his professional recognition. People should not be divided into those who have fulfilled themselves and those who have failed in the ability of self-fulfillment; rather, they must be situated on the path to self-fulfillment. Thus, it is also necessary to assume that the authentic self is a changing and developing variable and is not in a static, final, and exposed status, not among people and not in the individual over time. Only the free person, who acts without limitations, will succeed in knowing himself and creating his authentic self.

According to Nietzsche\textsuperscript{14}, it is not possible to leave the person being educated alone without a directing educator, and this so as to prevent a harmful and continuous influence of the social hegemony on the educated person. The suitable educator should propose to his pupils ways for authentic search and should pull the educated person to adopt and internalize significant themes from his teacher’s life and form of behavior. The educated person can be freed of these bounds and develop personal authentic creativity and thus increase in himself independent educational authority\textsuperscript{15}.

Many changes in education have occurred until today. According to Shwartz-Franco\textsuperscript{16}:

The pupils of today do not know the name of John Dewey. But each and every one of them – in the elementary school, the middle school, and the high school – must be grateful to him. For what? For art lessons and com-

\textsuperscript{15} E. Goskov, \textit{The Development of Educational Authority}.
puter lessons, for diverse teaching that includes research work, conversations, watching films, a yearly trip, the presence of counselors and assessment for learning disabilities, for all the choices and accommodations, for the personal attitude. All these possibilities, which appear to students and teachers to be obvious, are an outcome of the educational revolution that John Dewey, the father of progressive education, formulated and lead.

Dewey\textsuperscript{17} said about himself (with justification) that he caused a Copernican revolution in education. Copernicus, as aforementioned, caused the revolution in the perception of the universe, when he stated that our planet orbits the sun and not the other way around. Dewey caused a Copernican revolution in education and convinced that the educators and the curriculum need to revolve around the child, adjust themselves to the child’s tendencies and abilities, and not the other way around.

However, Dewey\textsuperscript{18} was not a simplistic thinker of ‘the child is at the center’; he attempted to balance the centrality of the child using social values from his democratic perception. Democracy, according to Dewey, balances between the individual interests and the general interests and in essence merges them. Dewey thought that in the democratic form of life the individual and society support one another and the prosperity of the individual ensures the prosperity of society and the reverse. On this background he established a school that involved the students in the planning and management. In Dewey’s school the pupils and the teachers participate in the life of the school community and strive to balance between personal goals and shared goals, between freedom and discipline. Dewey believed in education based on experience, education that offers the pupils possibilities to attempt considerations of knowledge and reciprocal relations. Children, according to his belief, learn concepts not through lectures but through experience. Traditional education, according to Dewey, is not built on children’s experiences and its abstract concepts do not connect to their experiences. From the children’s perspective, such abstract concepts lack meaning and therefore the school learning is boring and exhausting.

What characterizes the educational mood of our time is disappointment with two models – the ‘curriculum at the center model’ and the ‘child at the center’ model. On this background a new model, a third model, is steadily emerging in educational thought. The first model supports the framework, the second model supports freedom from the framework, and the third model supports freedom in the framework\textsuperscript{19}.

\textsuperscript{17} Ibidem.
\textsuperscript{18} Ibidem.
\textsuperscript{19} Y. Harpaz, \textit{Towards the Third Model}, Culture and Society in Education, 2006 June, 36.
Thinkers who came after Dewey improved the distinction he drew between ‘old education’ and ‘new education’. Lem\textsuperscript{20} divided education into three ‘logics’: logic of sociology, according to which the goal of education is to accommodate the child in the society in which he lives, the logic of acculturation, according to which the goal of education is to shape the child’s spirit in light of the values of culture, and the logic of individuation, according to which the goal of education is to enable the child to fulfill himself. The first logic and the second logic represent ‘old education’, namely, the first model, and the third logic represents ‘new education’, in other words, the second model in which the child is in the center and has the freedom from the framework.

The third model appeared on the background of the disappointment with the two older models. Indications of it, in thought and in deed, can be found in different places in the Western world. Frequently they can be found under the name ‘community’, for example, community of learners, investigation community, reflective community, community of practice, and thinking community. Hence its focus is clear: joint activity of teachers and pupils in order to solve problems, create knowledge, and understand the ‘world’.

From the perspective of the third model, the two previous models are not contradicting but are similar essentially in terms of the perception of the learning, the teaching, and the knowledge they embody. Rogoff, Matusov, and White\textsuperscript{21} explained that, according to the older models, there are two sides of learning, an active side and a passive side. According to the first model, the teachers are active: they lecture, they hand out tasks, and they test, while the children are passive: they listen, they do the tasks, and they recycle knowledge in the tests. According to the second model, the children are active: they manage the learning on their own, while the teachers are passive: they constitute mobile sources of information. In contrast, the third model removes the assumption that children and adults are found on the two sides of the fence; both are mutually involved in the joint investigation. Similarly, Bereiter and Scardamalia\textsuperscript{22} wrote that education must extract itself from its fluctuations between teaching of ‘the curriculum at the center’ and teaching of ‘the child at the center’. It is necessary to fashion a third way for


the management of education according to the model of the research group – the knowledge building community.

The Gordon College of Education in Haifa established its status as the leading college in Israel in the field of project based learning (PBL). In recent years an increasing number of courses on PBL in different and diverse knowledge domains have been added. This year I built and taught a first course in the college on PBL in mathematics for students for teaching mathematics – 27 students. In this course the students received four introductory lectures on PBL. In the six weeks after these lectures, the students attempted and taught mathematical skill in a heterogeneous group of four pupils in elementary school classes through projects. The lessons held in the college during the six weeks focused on fine tuning, a process in which I sit with each group of students separately and together we build and analyze the process, after the students learned the topic on the level of the individual student.

Together we wrote drafts and reflections, we changed and improved. In some of these lessons there were feedbacks in the joint sessions for students who reported the processes in the schools. In the last three lectures the students summarized the process they had experienced, addressed special events, difficulties, problems and their solutions, strengths, sensitive issues, and points for improvement. In addition, in every presentation the students addressed the three elements of PBL: summative event, drafts, and feedback and the six principles of PBL: authenticity, academic rigor, applied learning, active exploration, adult relations, and assessment.

The students chose this course rather fearfully, primarily since it is a different course, in the framework of which they must work in the school, teach the children mathematics through projects, and obtain feedback and support in the course sessions. The students were afraid about the course requirements, which included teaching mathematics through projects of pupils and writing a course journal that includes the description and planning of the project. The journal included description of the summative event or the constructed product, the drafts, the summaries of our different feedback sessions in the lessons and of the pupils in the elementary school classes, and reflections addressing the six principles of PBL: authenticity, academic rigor, applied learning, active exploration, adult relations, and assessment.

Furthermore, the students feared the pupils’ functioning in the project work process. They were not certain whether the pupils would connect to the work, enjoy their learning, be curious, be interested, cooperate, and succeed, both in the acquisition of the mathematical skill and in the process of the project and the preparation of the product.
Kaplan and Rafaeli\textsuperscript{23} examined the emotional and motivational processes and products entailed in the implementation of the PBL method among student teachers from different cultures, Jews and Bedouins. They found that in comparison to the students who learned according to the traditional method, the students who learned according to PBL reported more positive emotions, satisfaction, flow, and investment in the course and experienced more support of autonomy and efficacy, regardless of the difference between the different cultural groups.

As in the findings of Kaplan and Rafaeli\textsuperscript{24}, the students in the PBL course I taught in mathematics at the Gordon College of Education felt the ambiguity at the start of the course, the feeling of exploration in a fog, and unclear expectations. One of the images in their research used to illustrate feelings at work on the project was ‘surprise party’, and in my course one of the images was ‘going on a rollercoaster’.

As in the findings of their research\textsuperscript{25}, my students wrote reflections about the importance of the lecturer’s availability and ability of inclusion and empathy. The students spoke about the same difficulties: absence of resources in the school, teamwork and collaboration of children without previous experience, their lack of experience in this type of teaching, and ambiguity in the course structure.

All my students and most of the Jewish students in the research of Kaplan and Rafaeli\textsuperscript{26} believe that they will implement the method in the future in the schools. In contrast, the Bedouin students address the cultural barriers to the implementation, such as the forbidden intermingling of boys and girls, the ‘unnatural’ combination between children from different families, the objection to the use of the Internet in Bedouin schools, the custom of frontal instruction, and the lack of openness on the part of the school principals.

My students reported feelings of satisfaction, meaning, relevance, and enjoyment. They felt that they and their pupils had the possibility of autonomy and choice. They told that they worked with great cooperation, although the teamwork had to address many difficulties. Over time they learned to surmount these issues, both the students and their pupils.

However, the students related that they learned about their personal traits, such as degree of responsibility, their independence in learning, self-

\textsuperscript{23} H. Kaplan, V. Rafaeli, Project Based Learning and Emotional-Motivational Experience among Student Teachers from Different Cultural Groups (Research Report), Tel Aviv 2015.
\textsuperscript{24} Ibidem.
\textsuperscript{25} Ibidem.
\textsuperscript{26} Ibidem.
direction, self-regulation, efficacy, weak points, and leadership. In the feedback and reflections conducted with the students during the work on the project, the students told about learning about their personal traits and primarily about the discovery of abilities related to self-management and orientation in learning they had not known about beforehand.

The students worked on different projects, such as peer teaching for the class, construction of a work booklet and presentation on the topic of measures of length, data research, dream room, construction of an active wall on the topic of multiplication, construction of a practice booklet for classmates on the topic of multiplication, and creation of a book of recipes with concomitant mathematical problems.

The reflection of the students indicated the following points:

The first point is, Reinforcement of the Relationship with the Children

The students describe the teaching through the project as an exciting, significant, and important experience, through which a strong relationship with children is created, even with those whose voice is not heard every day. The student Tafet said: “The project allowed me to be in a more personal relationship with children who sometimes in the class cannot be heard. This window is very important to me in teaching, even more than the contents.”

Freedom in the Framework

The students’ reports on the children’s functioning during the work on the project indicate that they felt a degree of freedom in comparison to traditional learning. The freedom was expressed in a more liberated atmosphere, which included learning outside of the classroom and conversation in a small group. These reduced the dissonance and created a higher level of intimacy between the teacher and her pupils. The student Tafet said:

I felt that the freedom given to the children in the planning and performance is important despite its cost. Support and criticism are essential, but in my opinion it is necessary to allow the pupils to make mistakes, even at the cost of a product that is not the best, since you learn from mistakes. It is clear that this is not easy to do, especially because of the desire to help, and yet to allow the children to understand and to examine themselves the meaning or implications of certain choices they made or decisions they decided. All these are a part of meaningful learning.
Public Relations for the Project

The ‘marketing’ of the project to the schoolchildren and their teachers contributed to the increase of the pupils’ motivation, since they felt a sense of pride in the unit and wanted to prove their abilities. The student Sharon said:

It was important to do public relations work [market] the project so as to increase the morale and so that the school would know what the students are doing. The students hung throughout the school signs with the research question and since the lower classes do not know the concept of frequency we hung signs. What is the color most worn in grade 1? Class 5B is examining the color of the shirt that is most frequently worn. On the classroom door we had a sign: ‘In this class data research is performed. Frequency, relative frequency, and mean will be examined.

Answer to Difference

In the beginning, I described from my perspective as a mathematics teacher who has taught for twenty years the difficulty in providing an answer to the different populations in mathematics in a traditional frontal lesson. I present quotes from students to show how the work with projects provides the best answer.

According to the student Dikla:

I summarize the project as a resounding success, and the primary reason is – that everybody worked!! In the project everybody was active, everybody was enthusiastically moving between the classes and being so important in the collection of the data, like little researchers. Then even those who did not know how to build the tables by themselves eventually communicated with their peers in the group and finished the answers from one another and everybody, without exception, drew a diagram and solved the worksheet.

In the class where I am working there is a worrisome distribution, a small number of pupils are very good and strong, lots of average pupils, and a few who completely disappear. (I analyze their behavior as complete lack of confidence and not as laziness or suitability for an inclusion class as I heard in the school.) For all the disrupters – hyperactive children there was research activity in which they did not interrupt, in which they found themselves, they participated and were serious in their work. I truly feel that there was essential, positive, and enjoyable learning. … I saw the satisfaction and interest in the pupils’ eyes.

According to the student Noga:
Two pupils were especially prominent in the group. One had leadership skills and she took matters into her hands and saw that everybody would work in the order determined. The other was prominent because of his different viewpoint and his exceptional knowledge. When a number of pupils did not understand a question from the booklet, he found a different and unique way to explain to them so that they would understand.

According to the student Dikla:

The adjustment of the project to every pupil was undertaken with forethought in that we divided the groups so that they were built from weak and strong pupils. We were careful that every pupil would have his own role so that everyone would feel active in the project, one would collect the data in the transition between the classes, another would draw or color diagrams, a third would analyze them, so that each and every one would feel a sense of doing and success. In addition, every pupil in the group is required to fill out a table of frequencies by grade and to file it in the notebook so that everybody constituted a part of the discourse in the group. I went among the groups and when there was a pupil who as more dominant I tried to motivate the rest by asking questions.

Tuning

Processes of fine tuning occurred simultaneously both among the children under the student’s guidance and in the course lessons under my guidance. In the tuning process, the students reported their progress, their difficulties, and ideas for change and improvement. Along with them we examined the future steps.

According to the student Noga:

During the sessions, there were moments when I directed the students to a certain point but there were also moments when I did not at all need to intervene and they led themselves exactly to the point I wanted them to reach.

Difficulties

Many teachers do not have the daring to attempt this type of teaching, when they have no idea ahead of time about the contents, skills, questions, and difficulties they will face during the work in projects with their pupils. Teachers are accustomed to preparing lesson plans that are structured from the beginning to the end, when shifts from them and changes remain in the content realm and the skills they know. In this way, the teachers feel in control and are confident when they enter the classroom, both in terms of the
contents and skills and in terms of maintaining discipline in the class and the desired quiet.

Not many teachers will be willing to change the traditional and safe way, unless the school principal or head of the Ministry of Education or the district manager instructs them to do so. I believe that they are afraid to display lack of knowledge to their pupils, afraid of the reports the pupils give to their parents or to the school principal about their lack of knowledge in some topic or another. These teachers will be afraid to be mocked by their pupils, by other teachers on the staff, and by the parents and it is also possible that they fear their employment will be terminated.

In addition, these teachers are afraid of the independence given to their students and of the loss of their status as the absolute givers of knowledge in the class. These situations may cause disquiet in the class and the disruption of the customary balance. It is also possible that this imbalance and the constant noise do not suit certain students and will cause them suffering. These reasons do not cause the teachers to be enthusiastic about the change and the departure from the traditional ways.

The problems raised by the students included the objections of teachers, which were addressed by me through my request of them, lack of resources, discipline problems, and difficulty with teamwork and collaboration among the children. Principals did not allocate budgets to the projects and therefore the students had to invest their own funds, when possible. Certain children felt mistakenly that they were free of the class conventions, and thus from the beginning of the work on the project they attempted to disrupt and to act out. Over time, and with progress, the children learned to enjoy the change. Others found the teamwork and giving difficult, but they also over time discovered the advantages of collaboration and the enjoyment it entails.

In classes where the students were the teachers there was learning in groups that include all the children of the class, but in classes where the students would work one day a week, then a sample group was selected, and this created chaos and disappointment on the part of the children who were not chosen. Sentences such as “Why them and not me?” were heard. In addition, there were difficulties with the choice of the work time, so that they would not miss lessons that would be difficult to make up. Certain parents also did not allow their children to leave the lessons, and therefore by default they worked at the end of the day.

According to the student Reut:

In the project My Dream Room with the IKEA book, the problem that I needed to address was the ‘wise guys’. In the work booklet we offered the children the option of using the written deals offered to them (50% discount, buy one the second is half
off, and so on). Another option was choice and personal writing of a deal that suits them. These wise guys wrote “Sale – Everything Free”. I attempted to explain to them that there is no such thing as ‘everything free’, that stores do not give presents for free, that everything in the world costs money, and that they needed to change their choice. I asked that they would begin to choose products according to the budget and to examine estimates. I was certain that these smart-alecks understood – but no – they simply sat and chatted among themselves and when I approached them and asked them whether they changed one of them said “I won everything for free since I am the millionth customer”. I did not accept the answer. I went to the board and wrote in large letters ‘There are no free products”. I went one by one and erased for everybody on the page the possibility of choosing a deal and I asked them to begin to work or they would return to the classroom. I explained to them that there is a reason why they are here and they need to cooperate. Many children did not understand, they said ‘You can’t cancel a sale’. I had not managed to respond and the teacher who sat in the library with me told them ‘Stores can cancel sales whenever they want, and this is something that happens in reality’, and thus they calmed down.

According to the student Tafet in the data research project:

The stage of the preparation of the presentation was for me the point where the difference between the children was the most prominent, and the fact is that some of the children do not understand what teamwork is. The children became caught up in competitiveness and feared that the other group would copy from them, although I again clarified that they are one staff that divided up for the sake of efficiency. In the groups themselves there were some who worked more and some who did not contribute much to the conversation, and this bothered the rest of the members of the group. In this stage the fact that the children quickly went to the stage of the performance in the dialogue of the stage of the preparation of the drafts was prominent. At these moments I intervened more and I asked the children to draw or to explain to me what exactly they want to do and why on the pages. The fact that each one of the children wanted to collect the data by himself and wanted to process the date by himself while concentrating with himself the materials was also prominent. From my perspective, the feedback is performed throughout the project and it expresses in intentional statements as answer to questions I asked, but mainly also natural and intuitive statements of the children towards one another, whether towards the design or mistakes during the execution of the algorithm. The stage of the preparation of the drafts, the stage in which the children concentrated the data, processed, and planned the projects constituted for me a point where I learned many things about the pupils and sometimes things I did not know, not necessarily about the topic of the average. Thus, for example, I realized that child writes the number 1.5 in the following way 1/5. In other words, he changes the decimal point for a diagonal slash line. In addition, this was an opportunity to show how and whether the children have mastered the topic of fractions, since the data processing required of them to make use of previous knowledge. Another thing I was exposed to is the difference in the ways in which the children chose to process the data they collected. I emphasized to the children the difference between one another and the uniqueness of each one of them.
Thus, for instance, one boy chose a verbal representation of the data, including the numbers. A second boy prepared for himself a table in which he calculated the mean while presenting the data that he used for the calculation, and one girl chose to present just the exercise. I will note that after I presented to the children the difference some of them changed their manner of writing.

According to the student Tafet:

In my opinion, the fact that I did not insist with the children to obtain a neatly finished product and I ‘was satisfied’ with the product they produced is important. The children and the teacher need to meet the time limit and to understand that they must plan the time and the project according to this limitation. In addition, the conclusions according to which the group members need to divide in a way different from how they divided or to define for themselves another project in light of the experience they had are important conclusions or insights. The fact that the children understood that they had to plan their conduct according to limitations is important. At this point I could intervene more but I think that if I were to navigate the children in a way that ‘they would not learn firsthand’ these points then I would be denying them important learning. Thus, even if this project were not ‘perfect’ in terms of the performance, I think that some of the children learned and derived important insights. In addition, the children will participate in additional projects and I hope that this project taught and gave them tools for behavior in projects in the future as well.

Evaluation in the Domain of Mathematical Skills

In all the groups, the holding of a test of knowledge before the beginning of the work on the project created antagonism among the group members. In some of the groups the students gave up on the test when they saw the children’s grumblings and in others they did not give up and continued with the original plan to examine the achievements following the work on the project. In essence, the children expected another type of work, free of the indications of traditional learning, and when they received a test they were to a certain extent disappointed. The members of the group that were examined before and after the work showed an improvement in the achievements in the skills learned or practiced.

According to the students Coral and Nataly:

Before the work on the project we passed out a worksheet to the students who are taking part in the project, this was like a test. In the beginning they were very angry and said, ‘Why a worksheet, we thought that this project is something fun’. We explained to them that the worksheet is for us and we do not show it to the teacher and that it is of course without a grade. At the end of the project we gave them another worksheet, at a slightly higher level of difficulty, so as to examine their knowledge
Project Based Learning (PBL) as a Promising Challenge of Teaching Mathematics

After the learning and work on the project. The students were very enthusiastic about the worksheet and were happy to solve it as opposed to the first time when they were angry. One pupil told me ‘How fun, it is sure to be easy for me now’. Another pupil said, ‘Easy peasy’. The students showed willingness to solve the worksheet and no longer saw it as a punishment. It is possible to see that after the project the members of the group have mastered the different topics in which they had difficulty in the first worksheet.

According to the student Tafet, “I see that the children who participated in the project participate more in the class when the subject is taught and they evince better understanding of the topic.”

According to the student Ortal:

Throughout the project assessment was made of the understanding of the topic by the students, during the frontal lessons, and in the limited meetings, through the asking of questions, worksheets for the examination and preparation of the products. There is no doubt that this is a very important and essential principle in the learning process in general and in the project-based learning process in particular. It was important to us to examine the understanding in every stage, so as to examine the continuation of the project each time anew and not to lose the weaker students on the way or students who less connected to this way of teaching.

Evaluation in the Interpersonal and Intrapersonal Functioning Domain

The students expected to improve the pupils’ interpersonal communication abilities as a part of the learning because of the continuous teamwork and the collaboration that requires respect, sensitivity, tolerance, and consideration of others, for the promotion of shared objectives and outcomes. It is possible to see in all the groups an increase in tolerance, respect, and consideration. Furthermore, children who in the frontal lessons did not learn and even disrupted displayed awareness of themselves, self-regulation, and self-control and contributed to the teamwork and disrupted less. The intimate relationship with the student and the collaborative work in the small group contributed to the pupil’s sense of efficacy. The opportunity was created for different learning, for a new beginning, and certain children who had lost their faith in themselves found their place in it.

The students Coral and Natali said:

It is possible to see a significant improvement between the first session and the last session. The pupils in the beginning of the project did not agree for any one of the group members to touch their pencil case. Each one was very protective of his equipment, each one thought to himself, and when a pupil shared his thoughts, another pupil would burst into his statements. We talked with the pupil on the topic of
respect and cooperation; we explained to them that this is the secret of success in the preparation of the game. In the third session a significant improvement was apparent, the pupils no longer cared who touched their personal equipment, they respected one another, and when one spoke, the group members did not interrupt him. It was evident that they had succeeded in building the game stations through teamwork and cooperation.

According to the student Shani:

It was fascinating to see the process in the pupils, the brainstorming, the arguments, the compromises, the giving in, the creativity, and the imagination, which derived from their content world. It appeared that the process contributed to them in the personal domain, in self-expression, broadened the knowledge on the learned topic, increased motivation and willingness for learning. Since the project was undertaken in a small group, the feeling was that every student has room to express himself and to deliberate and to say his thoughts and emotions, as opposed to studies in the class, in the frontal method. It seems that during the work they get along better with one another.

The student Batel said:

The project required the children to work in collaboration to differentiate from competitiveness and I felt and saw that this was difficult for children. Both because they are not accustomed to this type of work and because of the interpersonal relations of the children. I am happy to note that I felt the change in terms of the point where we began the project in which every child hid his questionnaire, for example, as opposed to the stage of the data processing, in which equipment was missing and a girl went and brought for everybody, without even being asked to do so by some of the children. In addition, the statements of some of the children throughout the project changed and there was greater expression of the fact that there was teamwork.

**Authenticity**

The question that was asked in all the groups was as follows. What is mathematics in everyday life? Thus, from the beginning the different group members were oriented on authenticity in their work.

The student Efrat explained the authenticity of the project My Dream Room:

IKEA is the store in which the children will purchase the products and it is also a completely real store, which you can find all around the world. In addition, the prices that are written in the catalogue booklet we took from the store itself are the completely authentic prices and are not imaginary prices, and thus the children are connected to reality and they are given a way to evaluate money. They are put in budgetary proportion and wise consumption.
Student Summary of the Advantages of Active and Involved Learning

According to the student Hanan:

It was interesting to see how the children are those who plan, initiate, and produce. Thus their manner of learning is independent, thus developing creativity, responsibility, commitment, collaboration, motivation, and imagination. In my opinion, this method is especially successful since the pupils acquire tools beyond the studied subject, tools that they can use in the coping with life, such as, for example, in the academic studies, in employment, and in social life.

The pupils cooperated, were interested, and were enthusiastic about all that we did during the sessions. The pupils all the time would ask us, “Will we do another such session tomorrow?” and “I love coming with you for the project.” It was very fun for us to hear the praise; this only encouraged us to continue onwards.

According to the student Shani:

The sessions with the children let me understand again how important it is to develop and give the children the platform for creativity, freedom, with little direction, and primarily a listening era. How much the pupils know and want to leave the regular and fixed framework of the regular lesson, even if it is experiential for them.

According to the student Dikla:

We began to see products, to hear positive things from the students, to see the smile, the desire, and the enthusiasm.

Suddenly I understood that the entire topic, from the beginning to the end, could be taught in this way. It was an experience that empowered me personally, enabled me to experience another way of learning that I would not have seen if it were not for this course, and it caused me to think about how I want to behave as a teacher.

I summarize the project as a resounding success, and the primary reason is – that everybody worked!! … I saw the satisfaction and interest in the pupils’ eyes.

According to the student Hadar:

In the end, I am very happy that I had the option of again experiencing the PBL project and again in a positive manner. I know that every project necessitates investment on the part of the instructor, seriousness, and thought but alongside this the product is so important, instructional, and gives the pupil the sense of success that he so needs. That we all need.
Doing the project felt to me like a rollercoaster ride. One moment you are done and you think that you will never succeed in surviving the descent and climbing up and one moment at the top you think you have succeeded with all the difficulties and in essence only when you reach the bottom do you understand the scope and power of the trip.

According to the student Noga:

In general, in my opinion project-based learning teaches responsibility since the learning does not rely on and is not based on the teacher but only is helped by him and it is an exceptional tool for alternative assessment since the participating pupils show true desire to succeed and to satisfy and not only to answer the questions and get a grade. ... In general, project-based learning changes the ‘rules of the game’, the way of learning, and the view of the subject studied through the project and the participants’ way of seeing themselves. There is no doubt that over the years I will add additional projects and I will incorporate them in the curriculum.

Summary

After the first course of the development of initiatives and projects in mathematics, which the pupils began with ambiguity and completed with tremendous enthusiasm, I believe that it is very important to teach mathematics in the elementary school through projects. In this innovative way the pupil is allowed to learn freely, autonomously, and independently in a framework that obligates responsibility, rigor, investment, tolerance of others, and teamwork. The answer to difference exists, so that every pupil finds his place and expresses himself. The degree of curiosity, enthusiasm, and interest increase the motivation to work and to learn and create opportunities for the development of skills required in the 21st century. It is necessary to continue to examine the pedagogical efficiency of project-based learning in mathematics and to research its optimal development.

Much has been written about project-based learning as learning significant for pupils, but nothing has been written about it as learning significant for teachers. Project-based teaching and learning provide the teachers with professional autonomy, which enables them to follow after their educational desires. Moreover, it gives the teachers a new type of authority, an authority that is commensurate with our era. In project-based learning the teacher designs by himself the curriculum and guides and evaluates according to it. As an expert in the world of topics he teaches and as an expert in the ‘introduction’ of the pupils to this world, he receives the pupils’ esteem and be-
comes a significant figure for them. This is the educational revolution required today. I thank Professor Stanislaw Dylak my supervisor from the Adam Mickiewicz university, Dr. Orit Heller Hayon, and the students in my course.

BIBLIOGRAPHY


Kaplan H., Rafaeli V., *Project Based Learning and Emotional-Motivational Experience among Student Teachers from Different Cultural Groups (Research Report)*, Mofet Institute, Tel Aviv 2015.


---

Appendices

"Hot from the Oven" Reflections of My Students

• "When a pupil asked for a marker from one of the girls for coloring, she said: “No, it’s mine and nobody touches!”

• After two meetings, it was possible to see an improvement in the teamwork and cooperation of the pupils. The pupils shared with one another in drawing and in ideas and let one another interrupt each other’s statements.

• In the last session, a significant improvement was apparent. The pupils no longer cared who touched their personal equipment.

• The pupils respected one another and when one pupil talked, the group members did not interrupt and they succeeded in building the stations of the game with teamwork and cooperation”.

• "It was fascinating to see the process in the pupils, the brainstorming, the arguments, the creativity, and the imagination that derived from their content world.

• It appears that the process contributed to them in the self-expression, broadened the knowledge on the learned topic, and increased motivation and readiness for learning.

• Since the project was undertaken in a small group, the feeling was that every pupil has a place to express himself and deliberate and to say his thoughts and emotions, in contrast to studies in the class in the frontal method.

• The sessions with the children allowed me to again understand how important it is to develop and give children the stage for creativity, freedom of action with little direction, and primarily a listening ear.

• I saw how much the pupils know and want to leave the regular framework of the regular lesson”.

• "From the exchange of the experiences among the group members I again felt the enthusiasm for the project and the activity.

• We began to see products, to hear positive things from the pupils, to see the smile, the desire, and the enthusiasm."
• In the end, I am very happy that I had the option to again experience the project in PBL and in a positive manner.
• I know that every project needs investment on the part of the instructor, seriousness, and thought, but additionally the process and the product are very important and teach and give the pupil a feeling of success that he so needs, that we all need.
• Doing the project was like a rollercoaster. One moment you are at the bottom and thinking that you won’t survive the drop and the climb up, and the moment you are on top you think that you have succeeded in all the difficulties and in essence only when reaching the bottom do you understand the size and power of the trip you experienced”.

• The project obligated the children to work cooperatively, and not competitively, and I felt and realized that this was difficult for the children, both because they are not accustomed to this work and because of the interpersonal relations of the children.
• I am happy to note that I felt a change between the point we started the project, when every child hid his questionnaire, for example, and the stage of the processing of the data, when equipment was lacking and a girl went and brought for everybody without being asked.
• In addition, the statements of some of the children during the project changed and there was greater expression of the teamwork.

• On a personal note, I will add and emphasize again that the project allowed me to be in a more personal connection with children, when sometimes in the class there is no possibility of reaching them or hearing them in this way.
• This window is very central in my opinion in teaching even beyond contents learned in the project in an applied and experiential manner.
• I felt that the freedom given to the children in terms of the planning and the performance is important, although it has a price. The support and the criticism are essential but in my opinion the pupil must be allowed to make mistakes, even if the price is a product that is not optimal, since people learn from mistakes.
• It is clear that this is not easy to do, especially given the desire to help. However, in my opinion this is a part of significant learning – to allow
children to understand themselves and examine by themselves the meaning or implications of certain choices or decisions they made.

- It seems that the project had important meaning to pupils both in terms of the learning experience and in terms of the relationship between the pupil and the teacher and among the pupils themselves.
- I learned that the definition of clear goals and objectives and schedules are most essential for the existence and promotion of the project, but it is important that first they will be clear to the teacher and then they will be clarified and defined for the pupils.

- The summative event, activity on the active wall that the children prepared on the topic of multiplication and division, was attended by many pupils, teachers, the mathematics coordinator, and the principal.
- We could see the pupils’ pride when they explain to their friends the games, play them, make corrections when they do not do multiplication correctly, which is something that was a pleasant surprise for us.
- Our pupils explained about the process they experienced and the games they prepared (the goal of every game).
- We paid attention that there is wonderful cooperation on the part of the pupils who were not in the project; they played, praised, and gave their opinion for their friends who invested in the project and built the games.
- The principal succeeded in seeing from up close how pupils with difficulties can be ‘prominent’ and serious and invest. He was very impressed by the pupils and by the project itself. He praised the pupils for their work and us for the initiative and outcome.
- During the project and especially at the end we paid attention that the self-confidence of the pupils was strengthened. In other words, they began to believe in themselves more, in both scholastic and social terms. They were more cohesive as a group and were as proud as peacocks beside the active wall, and this amused us and made us happy.

- In the class where I am gaining experience, there is a worrisome distribution: a small number of very good and strong pupils, many average pupils, and a few pupils who completely disappeared. (I analyze their behavior as complete lack of confidence and not as laziness or suitability for inclusion classes as I heard in the school).
- In the project, everybody was active and enthusiastic about moving between the classes and being so important and collecting data, like
little researchers. Then those who did not know to build the tables alone in the end communicated with their group mates and filled in the answers and each one without exception delineated a diagram and solved the worksheet.

• All the hyperactive disrupters did not interrupt, they found their place in the activity, participated, and were serious in the work.
• I truly feel that there was essential, positive, and enjoyable learning.
• I saw the satisfaction and interest in the pupils’ eyes.

The pupils teach their peers, are active, and display curiosity, interest, and understanding.
The pupils plan their dream room
Project Based Learning (PBL) as a Promising Challenge of Teaching Mathematics
From the PBL exhibition in Gordon College of Education. Active & Involved Learning in Mathematics in the Elementary School – March 2016

My students and I presented the course: "Development of Initiatives and Projects in Mathematics".
We presented the projects they worked on like: creation of an active wall, the young author, room of my dreams, book of recipes, measurement of length, mean, etc..