Project-Based Learning (PBL) as a Promising Challenge for Prospective Mathematics Teachers in Math in Elementary School Education

October 2019

Supervisor: Prof. Stanislaw Dylak

Adam Mickiewicz University, Poznań

Ph.D thesis

Miriam Hilai

Acknowledgments

I would like to take this opportunity to express my heartfelt thanks to Professor Stanislaw Dylak, my supervisor, for his praiseworthy guidance. I also want to thank Dr. Orit Heller Hayon – the head of the Higher Studies program for her encouragement and support, Professor Hanna Krause-Sikorska – the Head of the Faculty of Education, Dr. Joanna Domagala – the manager of the external relations of the university, and all the staff at Adam Mickiewicz University in Poznań.

I also would like to thank Professor Yehezkel Taller – the President of Gordon College in Haifa. I want to thank the previous head of the Department of Mathematics, Professor Ilana Levenberg, who allowed me to initiate the course at Gordon College and encouraged me to explore it, as well as the current head of the department, Dr. Aliza Malek. I also thank Professor Ilia Sinistki, because of whom I love to teach and to research in mathematics, and Professor Leehu Zysberg, the head of Research Authority in Gordon College, who allowed me to research the course in the college.

I thank all the wonderful students who participated in the research study.

Last, I express my gratitude to my dear family, who supported me throughout.

Table of Contents

List of Tables

Abstract

Contemporary research studies teach that there is practical innovative learning, which is active and involving, called project-based learning (PBL). PBL provides a solution for the improvement of the performances in mathematics, for the motivation of the pupils, and for the inspiration of interest, curiosity, and enjoyment from this field of knowledge.

Five years ago, I initiated a course in the college where I teach, the Gordon Academic College in Haifa, called "Development of Initiatives and Projects in Mathematics". The objective of this course is to train students for teaching mathematics using projects. In the second year I taught the course, the research study was performed, and its results indicate the great value inherent in this method of active and involving teaching. From my experience as a teacher in the past and from the reports of my students in the Gordon Academic College for Education in the PBL course, in such teaching a solution is provided for the different populations in the class.

The pupils are engaged in learning in practical and realistic projects that are relevant to their lives. They are more active and autonomous, work cooperatively, and develop patterns of behavior of independence in learning, self-orientation, and self-regulation. These skills and patterns of behavior are important to their lives as adults and cultivate the six functions of the learner that are derived from the curriculum in Israel: sensory-motor, self-direction in learning and in its management, intrapersonal and interpersonal, and cognitive and meta-cognitive skills.

Introduction

Mathematics has always presented a challenge, both for teachers and for pupils, all around the world. Teachers of mathematics of all time periods are interested in causing their pupils to master the mathematical skills and to love math. They deliberate on ways of teaching, because of the tremendous gaps in their pupils' cognitive abilities and their non-uniform abilities to pay attention and to concentrate. It appears that the main solution in the frontal mathematics lessons is offered to the average pupils, but the main goal is to provide a solution for the entire population in the classroom. Fares,M.(2016).

Contemporary research studies teach that there is practical innovative learning, which is active and involving, called project-based learning (PBL). PBL provides a solution for the improvement of the performances in mathematics, for the motivation of the pupils, and for the inspiration of interest, curiosity, and enjoyment from this field of knowledge.

Five years ago, I initiated a course in the college where I teach, the Gordon Academic College in Haifa, called "Development of Initiatives and Projects in Mathematics". The objective of this course is to train students for teaching mathematics using projects. In the second year I taught the course, the research study was performed, and its results indicate the great value inherent in this method of active and involving teaching. From my experience as a teacher in the past and from the reports of my students in the Gordon Academic College for Education in the PBL course, in such teaching a solution is provided for the different populations in the class. The pupils are engaged in learning in practical and realistic projects that are relevant to their lives. They are more active and autonomous, work cooperatively, and develop patterns of behavior of independence in learning, self-orientation, and self-regulation. These skills and patterns of behavior are important to their lives as adults and cultivate the six functions of the learner that are derived from the curriculum in Israel: sensory-motor, self-direction in learning and in its management, intrapersonal and interpersonal, and cognitive and meta-cognitive skills.

Mathematical thinking has two attributes. The first is its layer nature, which means that mathematics, more than other areas, is built step by step. A mathematical argument is usually long and is based on the stages that preceded it and previous knowledge. The uniqueness of mathematics is its stable and strict connection between the parts of the argument and hence the commitment to accuracy, which is the second trait of mathematical thinking. To meet the complex demands and accuracy of mathematical thinking, strict thinking discipline is required. This discipline means that there is something more important than the researcher or the pupil and his desires. Thus, it teaches an important lesson, which pertains to the status of the individual person and the person in general in the world. Another issue that arises from the requirements of mathematical thinking pertains to the need for the investment of considerable effort, which bears fruit, and for the existence of a place completely different from the starting point[1].

Teachers of mathematics of all times deliberate about ways of instruction, the use of means of illustration, the nature of the assigned tasks, the heterogeneity of the class or the group, and the use of independent or collaborative work. These deliberations derive from their desire to cause their pupils to know and to love mathematics

It appears that the main solution in the mathematics lessons is primarily for the average pupils. Mathematics teachers, even the most talented of them, cannot plan frontal lesson plans that provide an answer for all the pupils, including pupils with difficulties and gifted pupils. When the inculcation of a new mathematics topic is held in about twenty minutes, the pupils with difficulties will not necessarily understand immediately and will need extended work. The excelling pupils and the gifted pupils understand within the first few minutes and become bored in the following minutes. Assuming that a number of minutes remain till the end of the lesson and a challenge problem is set for the talented pupils, there may not necessarily be enough time to engage in the different ways of solving it.

[1] R. Aharoni, Education in the Teaching of Mathematics, eds. Y. Tadmor & E. Freiman, *Education – Question of the Man*, 2, Mofet Institute, Tel Aviv, 2015.

The advanced technology and the available and frequent stimuli influence our children. The teachers who are at the front of the class appear in the children's eyes as boring and monotonous figures, who do not move but only change about every hour and a half, unlike the figures who change in the advertisements in the amazing time of fractions of seconds. The ability to pay attention is not equal among all the pupils and when the teacher teaches frontally, certain pupils are not focused on parts of the lesson, or even all of it, and therefore do not understand what is learned, and not necessarily because of learning disabilities.

The advanced technology and the gaps of attention deficit disorder disrupt the normal course of the lesson. Pupils with attention deficit disorder (ADD) stare and lose interest since they missed information that constitutes a basis for full understanding. Pupils with attention deficit hyperactivity disorder (ADHD) disturb the class because of their boredom, which is caused by lack of understanding. The teachers of the 21st century have to be different from the teachers of the past. They have more technological abilities, unlike their predecessors, but very often, they do not try to teach differently. They teach in the same traditional ways while complaining about their pupils' disorders.

Over the years I have searched for different ways beyond frontal and individualized teaching, so that I could provide a solution for populations with different needs in the mathematics lessons - pupils with difficulties, average pupils, excellent pupils, gifted pupils, and pupils with ADD and ADHD.

My search for alternative ways derived also from the need to promote the achievements and to boost the motivation, interest, curiosity, and enjoyment in the learning of mathematics. It is clear to every person who works in the field of education that it is important to give to each one of the pupils in the class an extensive, rich, challenging, and supporting solution in the different areas and in the different aspects - intellective (cognitive), emotional (affective), social and value-oriented, sensory-motor, and spatial aspects. The thinking about and search for ways to achieve this solution continue all the time[2]. My students who are earmarked to be teachers in a few years are

[2] J. Taran, *The Story of the Weave*, Efi Meltzer, Haifa, 2011.

more talented than me on technology means. They teach math with IPads and smartphones through Widgets. All the children seem to be active, and there are many smiles around. This is one example of an alternative way. Project based learning (PBL) is one of the alternative ways for meaningful learning and for the development of the learner, training him for the skills of the 21st century. As lecturer , after I discovered the attributes for this method of teaching, my role is to pass it on to my students, student teachers, for its continuation in the educational system.

I. The Search for Alternative Ways in the Teaching of Mathematics in the Elementary Schools

In this chapter, I will describe the different approaches in the teaching of mathematics over the years and the different populations in the mathematics lessons: the children who have difficulties, the average children, and the very talented children. I will describe their needs and new approaches for supporting them.

1. Approaches in the Teaching of Mathematics in the Elementary Schools

Mathematics and Ways of Teaching. Mathematics has always constituted a challenge, both for teachers and for students, all around the world. Teachers of mathematics of all times deliberate about ways of instruction, the use of means of illustration, the nature of the assigned tasks, the heterogeneity of the class or the group, and the use of independent or collaborative work. These deliberations derive from their desire to cause their students to know and to love mathematics.

Campbell (1997) asserted that the traditional teaching of mathematics focused on the teacher, who provides examples and explanations, sometimes with the use of illustrative aids so as to emphasize the intended meaning. In this traditional method, the children are required to look, to listen, and to practice.

This method generally emphasizes the ways of adults to doing mathematics. However, teaching that is defined primarily in terms of the teacher's actions and in terms of the students' practice assignments ignores an essential element - the children's thinking. What do the children think? Are the ways in which the children approach mathematics precise to the same extent and perhaps more significant for them? Are the children's ways mathematically mistaken or ineffective? Teachers need also to ask why children think as they do. If the children's thinking is not taken into account, then teachers will make decisions related to teaching without decisive information that can offer important insights.

From this thought, the National Science Foundation funded the IMPACT Project (Increasing the Mathematical Power of All Children and Teachers). The results of this project strengthen the idea that when teachers think in-depth about the way in which children understand and 'build' mathematical meanings, they can make decisions regarding their teaching and can organize their classroom in ways that encourage and support more meaningful mathematical learning, which are expressed in a considerable rise in the students' achievements. In addition, teaching for understanding yields the growth of children of all levels of ability.

The TIMSS Report (2007) emphasized the cognitive skills required in mathematics. The report presented three levels of thinking required in mathematics: knowing, applying, and reasoning. The first level, knowing, includes facts, procedures, and concepts. The second level, applying, is required for the solving of problems with the application of knowledge and conceptual understanding. The third level, reasoning, includes unfamiliar situations, complex relations, and multi-stage problems. As the student is capable of remembering more knowledge that is relevant and understanding a broad range of concepts, his ability to engage in problem solving and reasoning increases.

Knowing includes skills such as recall – the recall of traits, numbers, geometric properties, and symbolization; recognition – the recognition of mathematical objects, shapes, numbers and expressions; computation – the computation of four operations of arithmetic in whole numbers, fractions, decimal numbers, or integers, and measurement; retrieval –the retrieval of information from graphs, tables, scale, and others; measurement – the suitable units of measurement; and classification/order – the classification of order of numbers and objects according to their properties.

Applying includes the following skills: selection – selection of an action or method or effective strategy for problem solving; representation – representation of mathematical information in diagrams, tables, or any other way equivalent to the given mathematical relationship; modeling – the creation of an appropriate model, equation, or diagram for problem solving; implementation – follow up and execution after the collection of mathematical instructions; and solving routine problems.

Mathematical reasoning includes the ability to think logically and methodically. It includes analysis – for instance, sketching of a given unfamiliar body, generalization, rephrasing of the results in more general terms that can be implemented more extensively; synthesis/integration – integration of different mathematical procedures to reach the results and integration of the results to create another result; creation of relations between different components of mathematical knowledge or ideas and linked representations; and justification, justifying the confirmation or refutation of the argument through the reference to mathematical results or properties and solving of non-routine problems.

2. Populations with Different Needs in the Mathematics Lesson

Gifted Students in the Classroom. It is important that a solution is provided for students with different needs and abilities in the mathematics lessons. The population of gifted students is one of the populations that need a special solution. They have strategic thinking, multidirectional and multidisciplinary. They are curious, creative, and original, and they take the initiative. The difference in their thinking is not only quantitative but also qualitative, not only faster but also different. They have high ability to rapidly retrieve existing knowledge and to exploit it well. New knowledge that is acquired is linked immediately and easily to previous knowledge.

These children have a tendency towards reflective thinking, and they have meta-cognitive ability (thinking about thinking), which helps in processes of transference of thinking skills from field to field. Most of the time is dedicated to the collection of relevant information, to the filtering of irrelevant information, to a broad perspective, and to the planning of the way. They know to explain the way in which they reach a solution, dare to express ideas that are not routine, and to be groundbreakers. They stand firmly and flexibly in ambiguous and complex situations as well (Baroody,1998).

It is important to allow these children to partake in challenging and in-depth activity with abstract and complicated concepts, to progress at a personal pace while exerting partial independence under the teacher's direction, to acquire self-discipline, to take risks, and to develop tolerance of ambiguity.

It is prohibited to give them extensive repetitive practice, for they must not come to think that mathematics is not interesting, is easy, and does not require effort. In addition, they should not be pushed to move fast - an accelerated pace is not a substitute for depth.

It is important that these children be given fascinating activities for individualized work or work in groups in the format of investigation, which requires a degree of initiative and discovery. It is necessary to especially avoid activities in which the students are led, step by step, to a single solution.

It is important that the activity encourage reflection and mathematical discourse, promote understanding, analysis, and synthesis, and provide a solution for the different learning styles (Wilkins, & Oliver, 2006).

Baroody (1998) noted that a worthwhile task for gifted students is not built like a prescription but enables investigation and treatment in different ways and leads through an unfamiliar field, step by step, naturally, to mathematical discussions.

According to Sheffield (2003), students investigate problems in-depth, when they go beyond the solution to thinking about the rules, comparisons, and relations to other mathematical situations.

Students with Difficulties in Mathematics. Students have difficulties in mathematics since they do not remember what was learned in previous lessons, do not understand the concepts appropriately, and their teacher advances in the material rapidly without reference to them because of the large number of students in the classroom. However, it appears that those who have difficulties in mathematics act without motivation and without effort, avoid asking for help, and adopt superficial learning strategies such as repeating what was done in class and learning by rote, without new attempts. In this way they have less mastery over their learning, their difficulties become continuous, and their achievements decline.

Kurukkan (2015) adds that students attribute failure or delay in mathematics to an internal reason, which is stable and outside of their control, when in actuality this is a

controllable reason. His research indicates that the important step in the promotion of the students' efforts is to increase the interest in learning mathematics and to increase their self-efficacy.

Carpenter and Lehrer (1999) explain the rapid forgetting of the learned material in the absence of the deep processing of the material or the mistaken coding. If the strategy is only learning by rote, then the students with difficulties do not have the ability to connect a new question to what was learned previously. In their opinion, deeper learning strategies promote deep processing and thus long-term memory, and therefore to succeed in mathematics the students need to create rich and integrated knowledge. Sweller (1994) adds that a student who lacks supporting schema for the solving of problems will feel a high cognitive burden and will find it difficult to solve problems.

Some report that the student's progress depends largely on the teacher's quality (Mcber, 2000). Haimowitz (1989) notes that inadequate or inappropriate teaching is the cause of most failures in school. However, he focuses the point and states that a teacher can make mathematics easy only for those that feel it is an easy subject.

Among students with difficulties in mathematics, there is a gap between the desired achievement and the actual achievement. Sometimes a one-time decline causes a sense of failure, and this becomes a self-fulfilling prophecy, because of anxiety, excessive embarrassment, and lack of openness to learning that follow. These students frequently fail in the acquisition of basic skills because of the future low potential level, lack of adequate willingness, lack of readiness for learning, weakness in abstract thinking, or cognitive rigidity.

Burns (2007) notes that it is important to help these students link between mathematical ideas, so that the ideas will not be seen as disconnected facts (for example, the relation between percentages and decimal numbers and simple fractions). It is important to build their new understanding on the basis of their previous knowledge (for example, multiplication on the basis of knowledge of addition). It is important to remember that correct responses must be accompanied by the verbalization of their thinking, so as to determine whether there is mathematical understanding.

It is necessary to determine the necessary mathematical content (concepts and skills) and to support it (important to organize it and arrange it in continuous segments adjusted to learning). It is necessary to build priorities - which topic is truly essential. It is necessary to create a routine of considerable practice, support, and exemplification expected of the students, as well as finding a number of solutions for the given problem. It is necessary to again and again repeat the concepts, skills, formulas, to provide reinforcement for every success, even for the smallest of them, and encourage interactions between students and especially between the strong ones and the weak ones, so that "the strong up will pull the weak ones up".

It is necessary to express mathematical relations explicitly and to understand knowledge on the basis of previous knowledge and not on memory. It is necessary to encourage them to perform 'calculations ahead of time', an action that encourages speaking and develops mathematical vocabulary and thinking, and to perform calculations in writing, so that it is possible to follow up after their thinking (Burns, 2007).

3. Innovative Ways of Teaching Mathematics

Research in the field of mathematical education leads to a different approach regarding the learning of children and appropriate teaching (Fennema, Carpenter, & Petersen, 1989). After the researchers gave up the assumptions about the hierarchy of skills, and using the attempt to understand the knowledge of children as built and deriving from both inside and outside the school, they developed models of intervention and teaching that begin from the child's existing knowledge and afterwards are based on this knowledge so as to enable him to participate in advanced activities or activities of a higher order of thinking. Instead of beginning with a list of academic skills, distribution of formal tests, and catalogue of achievements and absences of the children, teaching based on ways of thinking begins with the recognition that children of every socioeconomic and cultural background and of all levels of ability come to the school when they have already acquired an impressive amount of everyday knowledge.

One of the examples of the approach to teaching mathematics based on ways of thinking is cognitively guided instruction, **CGI - Cognitively Guided Instruction.** This program for teaching mathematics in the elementary school classrooms integrates the findings of research studies on children's ways of thinking in mathematics with findings about the way in which teachers use this knowledge when they make decisions for teaching.

During the past eight years, the CGI staff accumulated a comprehensive corpus of knowledge about the development of the concepts and skills in addition, subtraction, multiplication, and division among elementary school children (Carpenter, Fennema, & Franke, 1992). This research study showed that even before children learned these concepts in a formal way, they can consistently solve simple verbal problems through examples, counting, or inventing methods for solution that are not related to traditional calculations in arithmetic. Children interpret and find logic in new knowledge in light of their existing knowledge. Therefore, experiences in solving problems, for the most part in the format of focused problems in a story context, which encourages the invention of solving strategies, constitute the basis of the development of basic arithmetic concepts and skills in the framework of this approach.

Most of the teachers with experience in teaching who made the transition from the traditional focus on skills and procedures to the CGI approach based on the solving of problems expressed similar feelings about CGI and learning disabled children (Hankes, 1996). As a part of an ongoing project of data collection, eight teachers of first and second grades who learned according to the CGI approach were interviewed and asked whether they believed that the CGI approach had influence on learning disabled children (Chambers & Hankes, 1994). One first grade teacher with eighteen years of teaching experience said, "A number of the learning disabled (LD) children succeeded very greatly with mathematics of CGI. I mean that they succeeded exceptionally. I assume that it is because they need to work hard all the time, so they know how to do this and how to solve the problems. If they can illustrate the problem in a direct manner, then they have it. And what this does for their self-esteem is wonderful! As they succeed more, they feel better with themselves." Another experienced first grade teacher noted, "The learning

disabled children can especially master verbal problems, perhaps not at the same effectiveness as others, but beforehand they never were there. You know, this is a great leap. They want to keep the pace and in this way they can. This lets them feel good about themselves." A second grade teacher openly expressed her thoughts about underachievers, "I discovered that in the past years the weak students always remained very weak and did not have self-confidence. I find that now they have more confidence, and I almost expect of them to be among the average students … I truly expected them to do all that everybody else does, and I find that they are doing it". (Hankes, 1996).

These remarks shed light on the most positive contribution of cognitive guided instruction – the belief that every learner has more mathematical knowledge than once was estimated. The message of CGI is that when teachers begin to listen to the children, they realize how much more children know than they previously identified. They realize that children have considerable mathematical knowledge that can be built upon. Teachers can achieve the objectives of compensatory mathematical education if they build on this knowledge.

Smith and Stein (1998) in the action research they conducted so as to improve the teaching of mathematics and the cognitive level required from the students saw that the choice and good presentation of a task on a high level does not ensure that the student will act at a high level. However, it seems that tasks from a low level never lead to activity on a high level, and hence the beginning with a good task is an essential condition for the employment of the students at high levels of cognitive thinking and reasoning. A good task includes, in their opinion, learning through repetition, procedures without relations to concepts or discipline, procedures with relations to concepts and to meaning, and mathematical activity.

According to the constructivist approach, which is prevalent today among educators, the learner builds by himself new understandings, new knowledge, and new perceptions. According to this approach, the person who learns is active in the structuring of knowledge and understanding and thus the starting assumption is that in every stage of the learning the student is an active partner in the building of the knowledge in the class.

The perception in which a learning process is performed when the 'teacher teaches – and the student learns' is not commensurate with the constructivist approach, in which "the learning process is the process of structuring, in which the teacher's knowledge does not have any superiority over that of the learner, since information is a product of personal structuring". Thus, the constructivist approach addresses knowledge as a product of the person's active building, which is done through the processing of observational information through existing conceptual patterns, in other words, through the linkage between existing knowledge and new knowledge to which the student was exposed (Smith & Stein, 1998).

Pitt and Kirkwood (2010) wanted to improve the mathematics achievements of their students through the use of nontraditional ways such as constructivism, in order to answer the needs of time. In their argument, the curriculum in mathematics does not need to come only from the Ministry of Education or the district but from daily planning that takes into consideration the needs of the students, teachers, topic, and content. In their eyes, the goal of mathematical education is to create an environment that supports the problem solving and investigation. They said that it is necessary to remember that this pattern is not suited to every learning environment. The observations of teachers and tests showed a considerable improvement in the learners' achievements and proof of the importance of constructivist teaching in mathematics as well as proof of the importance of teaching based on the student's previous knowledge and experience, encouragement of the discussion and guidance of the students to structure knowledge actively, and use of the students' ideas and their centrality in the class (Pitt & Kirkwood, 2010).

Many schools around the world teach according to the constructivist approach. The constructivist view of learning has received a great deal of attention in the last three decades, because of its impact on science and mathematics education. However, while many educators stressed that there is a difficulty in translating a constructivist theory of learning into the practice of teaching, constructivism helps inform teaching and reminds educators that the learner must be at the center of pedagogies (Alsharif, 2014). However, according to the research conducted in Riyadh in Saudi Arabia in 2013 it was proved that this approach is not suited to their teachers. It appears that teachers in Riyadh feel lack of

confidence in constructivist teaching, in which the student has prior knowledge that he uses. Moreover, in teaching in this way it is not possible to maintain absolute quiet during the lesson and this is an additional reason for the lack of confidence of the teachers in Riyadh. The teachers there give limited freedom to raise ideas of students but nevertheless they are certain in the empowerment of their students (Alsharif, 2014).

Another way to teach mathematics with pleasure is through games. In Argentina in 2010 a new model was developed, called "Mathematics for All" and characterized by the learning of mathematics through games. The model was examined and proved to improve the learning through the focus on the development of more mathematical thinking than the memorization of formulae. The research study was conducted among fourth grade pupils in weak regions in which there are families with special needs. Half of the teachers who participated improved their perception of their students in their reasoning ability, their leaning, and their work in groups. Their opinion did not change about mathematics as an area of knowledge and about their self-image as teachers of mathematics (Naslund-Hadley, 2011).

Meyer (2014) speaks about the advantages of the reciprocal teaching approach in mathematics for small groups of students for the solving of mathematical problems for the support of the understanding of solving word problems in mathematics. The reported approach is the dialogue approach to reciprocal teaching in mathematics that supports the learners in their closest fields through help of the teacher, support of the peers, and incentive cards. Use is made of the approach of the gradual release of responsibility in support of the students' independence.

The teachers illustrated explicitly every stage or reading strategy in the context of solving word problems in mathematics. They guided the students to implement every strategy through work in small groups and to gradually release the responsibility when the groups worked towards independence through each one of the stages of the process.

This approach encourages the development of a responsible discourse that can be explained and read closely and in-depth in the content of problem solving in

mathematics. The stages in the reciprocal teaching included: predicting, clarification, questioning, visualizing, connecting, calculating, and summarizing.

These stages were broadened and included the stage of providing feedback on the process, the minimal graphic organization of every stage, and interactive support journals. However, emphasis was placed on a broad and deep interpretation of cognitive strategies of illustration, connection, calculation, and summation. The approach of reciprocal teaching reported in this research helps discovery and supports students when they learn to perform more complex actions on a higher level. It makes use of support to teach cognitive strategies on a higher level and gives the students feedback, which is an important part of learning and teaching of these cognitive strategies.

Who Is the Best Teacher of Mathematics? In past decades there is also clear demand for expert and professional teachers in mathematics in the elementary schools. A recently conducted research study examined teachers with high levels of pedagogical knowledge and lower levels of content knowledge and mathematicians with low levels of pedagogical knowledge and high levels of content knowledge in terms of their instructional explanations. It was found that the explanations of the teachers and the mathematicians were different primarily in their process orientation. While the teachers presented primarily the stages of the solution of the problem (outcome orientation), the mathematicians also provided information to clarify why a certain stage is necessary in the solution (process orientation). It was found that students who learned with the process-oriented explanation had superior performances in comparison to students who learned with the outcome-oriented explanation in the test of implementation. The students who obtained only the problem without an explanation for learning displayed the lowest learning achievements. It appears that in-depth content knowledge helped the instructors create explanations with a high process orientation, a textual characteristic that served as valuable scaffolding for the understanding of mathematical processes for students (Lachner & Nuckles, 2016).

Copur-Gencturk (2015) examined teachers who improved their mathematical knowledge and analyzed results of multilevel growth models. The results indicated that

the achievements in the teachers' mathematical knowledge predicted changes in quality of the planning of their lesson, in their mathematical agenda, and in the class climate. In addition to the achievements of the teachers in their mathematical knowledge, it was apparent that their level of knowledge at the end had a significant role in the quality of the changes in their practices (Copur-Gencturk, 2015).

Technological Development. The current era is characterized by technological development. Hershkowitz and Biton (2014) lecture on interactive computer-based tasks and focus on the potential innate in their integration as format assessment tasks. Such assessment tasks enable us to evaluate conceptual ideas, to look at the process and not only at the final product, to help document the process, to follow up after the student's strategies, and to identify the students' mistaken perceptions. The information saved following the student's performances has an added value: it helps the teacher in the planning of the continuation of the teaching and helps the student not repeat mistakes.

One of the examples in their lecture addressed the numeral insight in vertical multiplication exercises. The activity offered every time a vertical multiplication exercise in which digits were missing. The missing digits were presented on the side, and the student had to pull each one to the appropriate spot. This activity included diverse tasks on different levels of thinking, for example, the completion to obtain the maximal/minimal multiplier and so on. The computerized system documented the student's performance and enabled the teacher to fully follow up after his progress in the task. They said that in comparison to the tasks of computerized assessment in which the teacher is not involved in the development and shaping of the task and accepts it as it is, the laboratory enables the teacher flexibility in the building of the tasks based on the laboratory and thus enables the adjustment of the task both to the present stage in the teaching and to the population of students in the class.

The use of the laboratory for the task of computerized assessment in the digital school environment has added value in a number of areas:

Integration between the printed media and the digital media. Since the laboratory is integrated in the relevant chapter in the textbook, it enables the teacher to integrate it in

the ongoing learning process and the laboratory becomes a tool of inquiry and active examination of hypotheses that derive from the nature of the task.

Look at the whole process: More than once we ask the student to answer a question, examine the correctness of the response, but do not succeed in following up after the entire process, which includes all the attempts that he made on the way. For instance, in the example of the parallelogram, let's assume that the teacher asks the student to state the minimal number of components that it is necessary to fix so as to obtain one parallelogram. In a paper and pencil test, we would receive a correct or incorrect response from the student, but we would not know the stages he went through until he reached the final response he wrote on the paper. However, his work process in the computerized laboratory is fully documented. Thus, the student can hypothesize at first that the length of the sides may lead to one single parallelogram. Examination of his hypothesis by pulling the parallelogram and the immediate response will lead the student to the conclusion that his hypothesis is not correct and he must search for an alternative strategy. Follow up after the documented process of the problem solving enables to see whether in one of the following steps the student will hypothesize that the fixation of the perimeter alone or the fixation of the perimeter and the sides of the parallelogram will cause one single parallelogram to be obtained. Of course, these three hypotheses are equivalent in terms of the solution, and therefore such attempts can indicate that the student apparently has difficulties understanding equivalence of the situations.

Use of different forms of representation. One of the significant advantages of the computerized laboratory over work with paper and pencil is the ability to see at the moment, through pulling, the degree of change of each one of the components presented on the screen. The added value of the different forms of representation enables us, as teachers, to ask questions that require different skills, which apply different levels of thinking adjusted to differences among the students.

Immediate feedback. The computerized assessment items enable immediate feedback for the student for his performances, which is not possible in traditional assessment of paper and pencil work. The computerized assessment items integrated in

the digital textbook enable the learner to continue to practice at a pace, time, and place suitable for him and on the basis of the immediate feedback to plan the continuation of the learning on the way to best performances, which is difficult to achieve in the regular class, in which some of the learners fear asking questions and/or sharing solutions.

To conclude, the computerized assessment tasks enable the performance of experiments and investigations, invites use of different forms of representation that encourage more interest, curiosity, creativity, and connection to the learner's world. They enable the evaluation of responses to non-routine questions such as the analysis of situations, the positing of hypotheses, and so on. Due to the technological instruments that act behind the scenes, these tasks enable the follow up after the learners' performances and reports on the level of the student, class, and grade (Hershkowitz & Biton, 2014).

II. Active and Involved Learning of Pupils as the Basis of Meaningful Learning

In this chapter, I will describe the need for meaningful learning and the different ways of active and involved learning of the pupils. I will extend on constructivism and on project based learning.

1. Meaningful Learning

A document of the Ministry of Education in Israel from the year 2013 that addresses the policy of the Ministry to promote meaningful learning began to analyze the event of learning Bible of children from age five in a place called the *cheder* (literally the room) in the period ranging from the Mid-19th century to the middle of the 20th century.

Yehoshua Sobol describes in his poem "With us in Tora Village" the social-cultural meaning that accompanies the beginning of the children's official studies in the *"cheder*". He describes this as a celebratory social event and a nearly religious ceremony. All the children on the street participated and the child began to learn reading by licking letters written in honey, so that the Bible would be pleasant for him.

The event contained main principles of meaningful learning. The learning was perceived as *valuable*: society and all the children attributed importance to the learning of reading and writing. The learning required *involvement*: the child, the center of the joyous occasion, was involved and active in a process that involves his body: he licks the honey letter. There was *relevance*: The event is linked to the needs of the five year old child and "The Bible was in his mouth as sweet as the taste of honey".

Our aspiration is for the pupils' learning to be meaningful, as sweet as honey. Our goal is for the educational process (learning-teaching-assessment) to ensure meeting the required achievements, alongside the sense of growth, value of efficacy, success, and self-realization, experience of discovery and solution for the pupils' curiosity, and to develop them as active people who fit into and contribute to society. For the process to

achieve its goals and become a lifestyle and school culture, we strive to enable meaningful learning alongside the educational continuum, while ensuring the balance required between orientation on the individual and commitment to society, between measurement and learning, between the process and the product, between teaching 'what' and 'how', between what is understood and what is not understood.

The Ministry of Education works to promote meaningful processes of learning, teaching, and assessment for the pupils, in a consistent and controlled process, which will contribute to a pleasant and growing experience of success, along with meeting the required achievements. "Today we are working on long-term strategic planning in a broad and participative course in which this perception will be expressed" (Cohen, 2013).

The document is found under constant structuring. "The discussion of meaningful learning is based on a number of fundamental assumptions on the nature of the person and the nature of knowledge."

- People are curious by nature. Learning and the search for meaning are a human need.
- Learning is a personal and interpersonal process. People learn in different ways and need space of choice and a feeling of freedom and control.
- People best learn when they accept upon themselves challenging but achievable goals.
- The integration between the acquisition of knowledge and the clarification of values, perceptions, attitudes, and emotions encourages meaningful learning and the cultivation of personal and interpersonal social abilities.
- Learning is developmental; education is effective when the learner's differential development and previous knowledge are taken into consideration.
- The learning occurs during experience with the material environment and the human environment, when a large part of it is performed in the framework of social reciprocal activity.
- People build knowledge or build upon existing knowledge, while relying on their current knowledge.
- People need constructive feedback and positive emotional climate so as to learn.

- Meaningful learning is based on three main principles: *value* for the learner and society, *involvement* of the learner and the teacher, and *relevance* for the learner. This learning offers the learner an interesting and challenging cognitive and emotional experience, which enables independent learning and learning in the interaction with others. (Cohen, 2013)

In the past, researchers assumed that teacher training programs have a negligible influence on the teaching of the teachers at the start of their path (Allen, 2009; Wideen et al., 1998) and that they tend to return to the traditional teaching methods (Cochran-Smith et al., 2010), although there is no agreement on the reasons: their profound beliefs on teaching or school cultures and processes of socialization or 'wiping' of what was learned or all together.

Storm (2015) proposes a conceptualization of teaching as an assemblage. The conceptualization reflects a flow of different components present in the school that influences the shaping of the teaching in different ways. The merger of elements in the teacher's story can be considered instructional assemblages, when each one influences different structuring of his teaching methods. However, according to her research teachers who primarily teach frontally because of behavioral problems in the class and because of other reasons attempt sometimes innovative ways that motivate and involve the pupils (Storm, 2015).

Mengel (2010) maintains that teaching must address cognitive, emotional, and spiritual intelligence, so as to enable learning that will be meaningful and important both for the learner and for the communities that the learners are a part of. Meaningful learning is supported by learning environments that encourage the pupils to create something meaningful, to have experiences of value with somebody or something, and to acquire new and significant approaches. The development of wisdom helps them learn the "ability to realize what has value in life for themselves and for others." (Mengel, 2010)

In this context, the results of the learning need to reflect personal growth and the ability to contribute to the communities of learners. Moreover, they need to serve as a

framework for continuous and consistent feedback and in parallel for assessment. In addition, the learning environment in general and the learning activities and evaluation opportunities especially need to be balanced around the Kolb Learning Cycle (1984). Beyond just the transfer of knowledge, the intention is the development of wisdom in meaningful learning for the solving of problems meaningful for the learners and their communities and the reference to the preferences for the different and personal learning of the learners (Mengel, 2010).

Harpaz (2014) in his article presents the question asked by Gardner, the father of multiple intelligences. What is the education that needs to be cultivated – the strong intelligences or the weak intelligences? Gardner's response was that the choice to focus on strong intelligences or weak intelligences is a product of value-based judgment and the two choices can be justified. In his opinion, at the beginning of life the intelligences should be developed equally. Over the years (during adolescence) it is necessary to focus on the stronger intelligences. In general, it is easier to go with strong intelligences and therefore it is necessary to set here a warning sign. "It is necessary to cultivate the entire range of intelligences!" To strengthen the weaken intelligences, it is necessary to dedicate more teaching hours and to use technological means.

Marion and Leather (2015) strengthen these arguments and describe an examination and assumption of responsibility for support of in-depth and significant learning through the creation of networks of districts that focus on similar objectives and similar effective strategies, which they identified in local internal processes. These district managers understood that it is not logical that the states will dictate to the schools the expectations of performances for the pupils and therefore they shifted the responsibility to the schools and to the districts during the interaction between them.

2. The Constructivist Paradigm

The constructivist paradigm, which is so often discussed in recent years, calls for the focus on the learning process. The forms of postmodernism in education, such as constructivism, represent an alternative method of teaching through the focus on the processes necessary for the understanding of the single pupil and the importance of the

role of the group in learning, with the strategies of teaching through research, the better use of situations in everyday life, and the contexts between them, and the teacher's new roles.

The learners need to address seriously and responsibly the learning environment that will allow them to be active participants. In other words, the learning influences the self-recognition and inspires motivation.

The emphasis is on the integration of a number of internal conditions and independent search, but also on the contact with the teacher, the group, and the computer. The pupil becomes a learning person who builds his knowledge through learning. The researchers take into account that every scholastic experience influences not only on the level of cognition and meta-cognition but also on the personal level, the models of approaches, influence on the self-recognition, and stimulation for motivation (Stefan & Popsescu, 2014).

3. Use of Technologies

eLearning is a genetic expression that describes a type of learning through electronic instruments. More precisely, the term describes learning that is performed using a computer that connects to the Internet, which offers the pupils, the students, and the course listeners a new opportunity to learn almost anything anywhere. eLearning can be defined as the synthesis obtained from electronics connects with teaching through the Internet. The term eLearning is also known as online learning and distance learning.

The advantages of eLearning technology include that it better addresses new needs of information and knowledge, a broad field of information sources is revealed for every type of educational activity, and they allow immediate self assessment and the reduction of the constraints of time and space.

The use of electronic tools leads to the development of skills of self-observation and identity. In all the topics in which the students improved their self constructivist learning, they exploited the advantage and special traits of the eLearning tool. The

computer helps autodidactic people find new information alone and complete missing information that came from other information sources. There is cooperation through correspondence with other users on the computer. In addition, use of electronic means through learning can develop meta-cognitive activity.

Stefan and Popescu (2014) noted that eLearning encourages the creation of constructive learning abilities since there is emphasis on the pupil's active participation in the direct process of knowledge acquisition. Eldakak (2012) holds that online learning eventually is learning that directs itself. Many formats were entered into these programs; however the pupil still sits in front of the computer by himself. The knowledge that he acquires or the advantages created in the development of self-esteem and increased earning ability depend only on the pupil himself.

Online synchronous teaching (Kear et al., 2012) is sometimes very demanding, alarming, and exhausting since it requires considerable concentration and focus. The teacher is required to hold verbal communication and written communication, to follow up after the participants in the discussion and in the interaction with everyone all the time, to help when technical difficulties arise, and to perform constant assessment. The teacher works not only with a variety of instrument online but also with resources and external programs. Teachers indicated that the technology of online discourse set before them a demanding environment that requires intensive effort and strategies for the encouragement of successful interactions lacking effective feedback (Cornelius, 2014).

4. The Ways for Meaningful Learning

Active and Involved Learning. Wasley (2006) maintains that the pupil's involvement is generally according to the pupil's abilities but if the goal of education is to encourage continuous learning then the educators need to involve the pupils in the class so as to encourage their leaning also in the universities. Kuh (2006) holds that the pupils' involvement influences their remaining in the system. Through the retention of the pupils in higher education, these pupils become life-long learners in the future and thus assure the survival of civilization. This learning can be achieved from the

assumption that the pupil requires knowing, understanding, self-perception, previous learning experience, willingness to learn, tendency to learn, and motivation to learn.

Caruth (2014) proposes a model of learning in which the pupil is involved in learning that includes:

- It is necessary to create a secure environment for learning, environment that encourages questions without criticism, which is secure, successful, and interesting for the pupils.
- It is necessary to ask the pupils about their fields of interest, so that they will be more interested in the class activities and the learning will be more effective if it is suited to their certain need for learning.
- It is necessary to help the pupils learn about the value of learning in terms of the perceptions, theories, and abilities.
- It is necessary to provide opportunities for learning in real life.
- It is necessary to present opportunities for cooperation such as group activities and discussions, tasks of problem solving, simulation exercises, and event researches.
- It is necessary to ask of the pupils to evaluate the learning. To share the responsibility for the assessment of the learning and the planning of logical learning objectives and goals for the pupils. It is necessary to help them be pupils who direct themselves.
- It is necessary to share the experiences of the attitude to the six points of the model, which will lead to the increase of the likelihood pf the pupils' involvement.

Warman (2014) proposes the Socratic method so as to involve the pupils in their learning. He maintains that it is necessary to analyze the importance of the presentation of differences in the field of citizenship as issues that are not resolved, so as to lead pupils to a situation of Socratic confusion, repeated clarification of basic assumptions, and encouragement of the transformation of pupils into thinking citizens.

Encounter with Cognitive Conflict in Technological Education. According to Autio (2009), many models of changes in the curricula in technological education exist today in the literature and in the textbooks. However, there is still over-emphasis of

passive learning and old traditions of the learning of the arts. The materials, techniques, and technology developed rapidly but the pedagogical content, according to Autio (2009), is limited only to the product. He maintains that it is necessary to develop the teaching of the learning material so as to improve the balance between the product processes and the generic processes such as motivation, investigation, planning, design, creation, and evaluation.

According to the model of meaningful teaching, the greatest problems are found in the stage of internalization. It is difficult for pupils to fix issues that were neglected in a later stage of planning, since the flawed mental image makes the action in the stage of the implementation more difficult.

Autio (2009) proposes that it is necessary to create internal motivation already in the early stages through an encounter with a cognitive conflict, even if this appears more difficult than the creation of motivation on the basis of outside factors. However, Ryan and Deci (2000) assume that external motivation can change into internal if the project is sufficiently interesting. Moreover, brainstorming, non-routine activities, high level thinking, and creative problem solving will be practiced already in the lower classes. When the pupils begin in the planning stage with beneficial and relatively simple applications, their motivation is increased.

Learning through Improvisation. Improvisation is not ‘shooting from the hip’ but rather the creative coping with the uncertainty that reigns in the rehearsal room in theater, in teaching in the class, and in life in general. Shem Tov (2015) maintains that the model of improvisational teaching may be the key to coping with this complexity, which is urgent and seeks a response in the training of teachers in particular and in the act of teaching in general. In his book he presents a model that processes the technique of theater improvisation, known to the audience from the program “Whose Row Is This?” into guidelines and rules for teaching in the classroom. Thus, there are more than a few examples and teaching books that illustrate its implementation and are explained through the known Shakespearean metaphor “All the world's a stage, And all the men and women merely players”. Alongside the practical aspect, the model corresponds with theoretical

concepts in the research of education, sheds light on them, and reveals their connection to improvisational teaching, such as experience, reflection, intuition, creativity, and critical dialogue.

Learning through Video. Hakkarainen and Vapalahti (2011) examined learning through video in contexts of project based learning and saw that in this learning there was something illustrative and authentic. This supported most of the traits of meaningful learning for problem solving. The self-reporting of the pupils about their emotional involvement was positive: enthusiasm, happiness, interest, and sense of community were the most common and intensified emotions. This was an encouraging outcome from their perspective since positive emotions predict high academic achievements.

Contextual learning is helped by learning tasks that are either situated in real and meaningful tasks or in a simulation through an event-based learning or problem-based learning environment. However, it is necessary to refine the learning task that the pupils seek to perform after they observe video clips. It is necessary to dedicate more time to writing and support of the cooperative and dialogue traits of meaningful learning and to provide more opportunities for cooperation and discussion.

Concealment and Visual Support – Cache-Cache Comparison

Wang and Fujino (2015) suggest hiding some parts of the information at the first stage of learning and then encouraging learners to actively detect them in the second stage. This process involving discovery in learning is defined in this research as "cache-cache comparison", coming from the French word for 'to hide' and bringing to mind the known game of 'hide and seek'. This is a system of support for learning that combines the advantages of meaningful learning and discovery learning by providing visual support and guidance when preventing erroneous perceptions and reducing the cognitive load. The method is intended to support the effective structuring of the learner's knowledge framework and is supposed to prevent the diminishment of curiosity and readiness to examine accompanying knowledge through the encouragement of active engagement (Wang & Fujino, 2015).

Games. Pavements in the school yard are painted with numbers, and the children jump on them enthusiastically. However, what appears as random jumps on these and other walkways are in essence the children's answers to mathematical questions asked by the teacher. The second grade pupils jump to the results of the multiplication and division problems, while eighth grade pupils jump to the numbers that represent the answers to questions on power and fractions and the upper grade pupils only tread on prime numbers and return via factorable numbers.

This game seeks to create among the pupils the connection between fun and enjoyment and the studies of mathematics, a subject that frequently is perceived as threatening and pressuring. Many research studies show that the transfer of mathematical topics in an enjoyable manner, which encourages and inspires curiosity, brings them closer to the subject and prevents difficulties in the continuation of their studies (Sharf, 2015).

Sharf (2015) quotes Dr. Yehuda Ashkenazi, from the Department of Computer Sciences and Mathematics at Ariel University in Israel, who conducted a two year research study, in the framework of which he developed games and activities for the enhancement of the thinking and the reinforcement of the mathematical knowledge. "Research studies show that the study of mathematics using play especially helps the weaker pupils, since it reinforces in them the self-confidence. The strong pupils manage in any event." Ashkenazi (in Sharf, 2015) further says that:

> But because of the play we can obtain a longer period of time of concentration from the weak children, and they are willing to invest additional time to learn what they must. In special cases of pupils with problems of attention deficit disorder, play is almost the only way to reach them. The strong pupils enjoy it no less than do the weak ones and although from their perspective such activity is not always beneficial didactically, it contributes to them in the reduction of the pressure.

Despite all the advantages of these activities, Ashkenazi holds that it will not be correct to use them in an exaggerated manner. According to him, experience also shows that these activities should not be performed regularly. The recommended frequency is a

short activity, about half an hour at the end of the lesson, once every two to three weeks, along with one long activity once a year. However, the optimal frequency depends on the teacher's feeling and the pupils' age. At a young age, the frequency of the activity will be higher.

Many research studies emphasize the risk in violent computer games and the increase in the level of aggression of children who play them. Many children play and enjoy them but the violent content may cause serious problems (Uhlmann & Swanson, 2004).

There are many educational games on the Internet and educational television programs. They can help the children learn spelling, arithmetic, reading, and other topics and can increase their interest and motivation in the school and their skills of mouse and keyboard use.

These games may also detrimentally influence the children in mental and physical terms because of the violence, and therefore Nachimuthu and Vijayakumar (2011) propose to set time limits on the educational games. It is important to them for the child to connect with the surrounding world and dedicate time to friends and family. In addition, it is necessary to explain to the pupils that games describe an imaginary world and not real life. They found that suitable computer games can serve teachers in the teaching process, so that the learning becomes more meaningful and promotes the children's knowledge and innovative skills in the scientific learning environments. However, the teachers are responsible for deciding about the most suitable game in terms of the level of the class and the curriculum. They must examine with care the contents in the games so as to prevent negative influences that will prevail over the positive influences.

To conclude, in Israel in recent years people have addressed the topic of innovative and meaningful learning and alongside it the topic of innovative assessment. Sharf (2014) notes that the Israeli Center for Innovation in education believes that the correct and methodical collection of data on pupils can enable the teachers to examine the results of the pupils' tests from different perspectives and according to different themes

and to identify gaps and weaknesses of specific children. Accordingly, it is possible to prepare focused work programs that will facilitate the closing of the gaps between the weak pupils and the other pupils in the class and to promote the excellent pupils.

The tools of measurement that the processes of methodical data collection implement were presented by the Center for Innovation by Columbia University in the United States and were adapted to the Israeli educational system. Dan Porterman, the head of the Israeli Center for Innovation, explains that the goal of the venture is to develop a culture in which the data lead the process of teaching: "The data are an effective tool for the principal and the teacher." (in Sharf, 2014)

> We see the pressures created in the schools following the Meitzav[3] tests or any other critical test but in the end the Meitzav is a good example of a test that does not serve the teachers as an instrument since they receive the data only in the following school year. At the stage when the results are in their hands, it is too late to hold the data that arise from it so as to implement them in the class. (in Sharf, 2014)

Porterman explains that the teachers who make use of the computerized measurement tools learn to use the data in proportion and primarily to rely on their personal judgment and on the information and personal relationship with the pupil, to hold personal follow up after the pupils, and to document the progress of each and every pupil. This requires of the teacher to use follow up after each and every pupil throughout the year but also averts for the teacher a situation in which he realizes only in the middle or the end of the year that the child is found behind his class. This can also provide a tool for the teacher to examine herself 'I attempted to do this and this and the results show that it did not succeed, so I need to search for another way.'

5. PBL – An Innovative Way for Meaningful Teaching

In project-based learning the project defines the final product for which the learning is intended. Holding an exhibition, building a device or structure, writing a play,

[3] The Meitzav tests are a system of tests conducted in elementary and middle schools in Israel in the subjects of science and technology, native language (Hebrew or Arabic), mathematics, and English. The word is an acronym meaning Measures of Efficiency and School Growth, and the word itself means positioning. The tests are held every year in one-third of the schools, so that every school participates in the Meitzav once every three years.

going to demonstrate, teaching a lesson, holding a debate, editing a film - all these and other activities can motivate the learning process towards their accomplishment. The project, in contrast to the question and the theme, also requires imagination in an early stage in the learning, which is expressed, for instance, in the question of "what will the final product look like?". In a network of schools in San Diego called High Tech High project-based learning is customary; and thus the science teachers teach about the circulatory system through calling the public to contribute to the blood bank and teachers of mathematics develop a casino in which the students are required to explain the different probabilities (Rahm, 2011).

Rahm (2011) added that project-based learning was invented one hundred years ago but is more suited to our era. In San Diego they developed a special version of project-based learning, and driven by its inspiration a local version was developed in the city of Holon in Israel. At the center of the method are the teachers: "If we give them autonomy, they motivate to other learning. The policy of the Ministry of Education is equivocal: it still does not help, but does not stop." He asserted that project-based learning is possible and desirable.

Much has been written about project-based learning as learning significant for students, but nothing has been written about it as learning significant for teachers. Project-based teaching and learning provide the teachers with professional autonomy, which enables them to follow after their educational desires. Moreover, they give the teachers a new type of authority, an authority that is commensurate with our era and functions effectively. In project-based learning the teacher designs by himself the curriculum and guides and evaluates according to it. As an expert in the world of topics he teaches and as an expert in the 'introduction' of the students to this world, he receives the students' esteem and becomes a significant figure for them. This is the educational revolution required today (Rahm, 2011).

A research study conducted in a university in Malaysia in 2012 compared between two groups of students. One group studied in the conventional method, while the other group studied using project-based learning. A significant change was apparent in

terms of the performances in the favor of the group that studied using project-based learning. This change was made possible because of the characteristics of project-based learning, such as cooperative learning, learning in small groups, discussions in groups, the presence of the teacher for help in learning, and resources for problem solving (Tarmizi & Bayat, 2012).

Another qualitative research conducted in South Africa examined during a year the project-based learning approach. The findings indicated that the exposure of the students to teaching in this way advanced in them significant patterns of learning that are characterized by the processing of topic critically and by processes that aim at independent learning and self-regulation over time. They used planning, problem analysis, product testing, adjustment and reflection of solutions, and metacognition. It seems that the students were less dependent on the lecturer. However, it was clear that there was tension on the matter of the change to the new approach. Some students insisted on obtaining correct and precise answers in the lecture and not through their experience (Malan, Ndlovu, & Engelbrecht, 2014).

Kean and Kwe (2014) note four components in meaningful learning: orientation in learning, active learning, constructivist learning, and authentic learning. In their research, through a blog that describes the students' experience in a course engaging in culture, it appears that the students see the components of constructivist learning and authentic learning as components with higher presence in meaningful learning.

There are cognitive, social, and instructional forms of 'presence' that exist in the meaningful learning environment. The cognitive presence addresses learning in which the pupils cannot shape their understanding and knowledge through an active relationship and communication in the learning process. In addition, the cognitive process can help in the creation of a challenging learning environment that can obligate the pupils to optimize their cognitive potential in the treatment of the learning tasks and solving of problems. The social presence addresses the way in which the pupils express themselves in social and emotional contexts. The findings hint that project based learning (PBL) is a

valid and effective alternative for the learning of the culture not only for pupils but also for their teachers who speak a foreign language.

Since the subjects in the research study are teachers of Japanese at the end of their training program, this cultural learning program will influence them as teachers when they teach their culture (Mahoney, 2009). The research found that the teachers' approaches, beliefs, and past experiences influence their teaching.

As Embi and Abdullah (2009) determined, the importance of the teacher in the creation of a meaningful learning environment should not be defined. Important tasks for the teacher in the meaningful learning environment are planning and organization of the tasks, motivation, and proposal of support for pupils and assurance that the environment leads and is suited to meaningful learning. However, the findings about the limited presence of active learning in contrast to the considerable learning of the presence of constructive learning (in other words, in response to the teacher's questions) hints that in the planning of project based learning it is possible that the teacher motivates 'active learning' through the presentation of content questions so as to instruct learning in the 'correct' direction in terms of the learning objectives. They propose that it will be more meaningful if the teacher does not attempt to 'govern', 'direct' or 'instruct' the learning too much.

In this context, Saluveer (2004) noted that despite the recognition of the importance of the learning of culture in the language lessons it still is limited primarily because of the lack of time, the choice of the topics, the learning materials, and appropriate activities and techniques. An attempt is made to overcome these difficulties through shared efforts of teachers in the planning and giving of experiences of the learning of culture, through the use of PBL and technology when the pupils examine and seek knowledge according their needs and areas of interest without dependence on a teacher who will provide this knowledge. in Sharf, 2014)

PBL - project-based learning - develops skills suitable for the 21st century. Today the educators understand that they need to help the pupils leave the school with skills suitable for the 21st century, such as cooperation, creativity, teamwork, problem solving,

and making decisions, so that they can learn, work, adjust, flourish, and succeed in an unknown future (Cervantes, Hemmer, & Kouzekanani, 2025). One of the ways to develop these skills is through Project-Based Learning – PBL.

The roots of PBL lie in the beginning of the 20th century, in the idea of progressive education formulated by John Dewey with his pupils. Dewey (1969) sought to connect between learning and life, the pupils' experiences, and their social needs. There is evidence that such learning is found in academies of architecture in the 16th century (Knoll, 1995).

Many educational experiences for progressive learning were performed from the 1920s around the world, such as in about one-quarter of the schools in the United States that worked in these methods and in hundreds of schools in Israel in kibbutz education and in the workers' sector. Nevertheless, educators did not succeed in bringing the perception to an educational routine because of the tremendous time of investment required of the teachers and the requirement for scholastic achievements and standardization (Rahm, 2011).

Assumed Results of PBL Teaching. From the beginning of the 1990s, the engagement in knowledge has become the main component in the work of people in the Western world and in parallel the technological advancement has added the dimension of access to knowledge. The skills for engagement in knowledge are interwoven in most of the new subjects in all areas of life and include collection, filtering, re-organization, creation, and presentation. In addition, the need for criticism and construction of meaning in the encounter between different narratives has become the need of the hour, with the extension of the perception of post-modernism that penetrated into most areas of life (Rahm, 2011).

Adjustment to the Present Era. The heterogeneity of the classes is accepted and more understood because of the psychological research of the period. The Multiple Intelligences Theory of Gardner (1996) and the Emotional Intelligence of Goleman (1997) extended the need for diverse holistic and unique ways of learning that education is required to develop alongside the intellect and constructivism. The constructivist

theories developed from the perception formed by Piaget and furthered by Vygotsky (2000) and others and maintain that people structure meaning actively and on the basis of previous knowledge. The constructivism has become the accepted learning perception and following it new perceptions of teaching and learning have developed (Perkins, 1995). Harpaz (2015) calls these perceptions the 'third model', which is mainly freedom in a framework.

Rahm (2011) maintains that for the educational system to be relevant to the 'third wave', a real cultural change in the classroom is necessary that will make it a community of learners. This change includes dialogic relationship – a productive dialogue between the teacher and the pupils, among the pupils themselves, and between the pupils and the knowledge (Hecht & Rahm, 2008) and relevance to the community and the environment – value-oriented learning related to topics, problems, dilemmas, and main themes. In addition, there is equal inclusion between pupils, the expression of a wide variety of areas of strength and growth, and the proactive handling of knowledge of the pupils, to differentiate from the learning by rote of the closed learning materials given by teachers and the shaping of organizational priorities, the organizational axis and the class space, so that they will support another learning process in which the teachers speak less and the pupils are more active.

The continuum of learning proposed to the teachers is not mandatory, but it brings to mind the aspects significant in the process that should be considered in the planning of the learning constellation. The areas can be topics or themes. The organizing axis needs to be semi-structured, or in other words, clarifying direction but open to changes. There is an intuitive presentation of the topic, exposure to initial knowledge in the field, initial feelings of pupils, initial questions that arise, conceptualization of intuitive ideas, and establishment of basic elements. In the continuation, there is brainstorming and thinking maps – the disassembly of the topic to questions or small topics: facts, ideas, learning topics that should be learned, and tasks that should be performed.

After this, there is the division into group and there is work in a group on the basis of the planning of times and resources and inquiry and collection of information, when in

part the information is brought by experts outside of the school. The teachers create in-depth learning and connect to the required skills, the required learning contents, the relevant social problems, and the content experts. There is an intermediate assessment, regarding the progress and pace, there is the continuation of the collection of information, the preparation of the final product and the summative event that includes presentation, and the reflection on the process they experienced (Rahm, 2011).

The San Diego PBL Model. In project-based learning the project defines the final product for which the learning is intended. Holding an exhibition, building a device or structure, writing a play, going to demonstrate, teaching a lesson, holding a debate, editing a film - all these and other activities can motivate the learning process towards their accomplishment. The project, in contrast to the question and the theme, also requires imagination in an early stage in the learning, which is expressed, for instance, in the question of "what will the final product look like?". In a network of schools in San Diego called High Tech High project-based learning is customary; and thus the science teachers teach about the circulatory system through calling the public to contribute to the blood bank and teachers of mathematics develop a casino in which the pupils are required to explain the different probabilities (Rahm, 2011).

Rahm (2011) added that project-based learning was invented one hundred years ago but is more suited to our era. In San Diego they developed a special version of project-based learning, and driven by its inspiration a local version was developed in the city of Holon in Israel. At the center of the method are the teachers: "If we give them autonomy, they motivate to other learning. The policy of the Ministry of Education is equivocal: it still does not help, but does not stop." He asserted that project-based learning is possible and desirable.

Much has been written about project-based learning as learning significant for pupils, but nothing has been written about it as learning significant for teachers. Project-based teaching and learning provide the teachers with professional autonomy, which enables them to follow after their educational desires. Moreover, they give the teachers a new type of authority, an authority that is commensurate with our era and functions

effectively. In project-based learning the teacher designs by himself the curriculum and guides and evaluates according to it. As an expert in the world of topics he teaches and as an expert in the 'introduction' of the pupils to this world, he receives the pupils' esteem and becomes a significant figure for them. This is the educational revolution required today (Rahm, 2011).

The High Tech High network was established in the year 2000 and it includes eleven unique schools in the San Diego, California region. These schools accept pupils by lottery. The network built its school culture on a curriculum that is based on project-based learning. Considerable thought was invested in the curriculum design. Its founders decided that they would not measure their pupils' success according to their achievements in different tests and measurements but according to the rate of their graduates who learned in the University. Since its establishment, 99% of the High-Tech High pupils continued their studies in a college or a university. 35% of them are the first generation in their family to complete academic studies.

In 2008 two organizations, the Paul Hamlyn Foundation and the Innovation Unit, launched the Learning Futures Project. Their objective was to find ways to improve the educational work in the high schools through the increase of the pupils' involvement in the learning. The project operates today in more than forty schools where the teachers develop innovative methods for teaching and active learning. The experience of the Learning Futures people shows that project-based learning and research-based learning, which are well designed and planned, strengthened the pupils' involvement in the learning and led to positive outcomes (Patton, 2012).

Three Important Elements of PBL. Multiple Drafts, Feedback, and Summative Event. Patton (2012) maintained that when pupils know that their project will be presented to people outside of the classroom, then something changes in the way in which they work on it from the very first moment. They understand that they will need to stand beside their work - in both senses of the word - to provide explanations and to answer the questions of their peers, family members, and even strangers.

This fact inspires in them greater motivation and commitment than does the desire to ‘get a good grade’. In addition, the event constitutes an opportunity for their family members, as well as for people from the wider circles of the community, to see from up close what happens in the school walls and to take part in the school experience, which leads to the reinforcement of the relation between the school and its community.

Multiple drafts have considerable value, even when the aim is to evaluate the personal work of each and every pupil, since they provide an excellent means for the teacher to understand the pupil’s process during his work on the project. It is possible to see the starting point, his progress and improvement from the first draft, through the following drafts, until the final product. This perspective is important for all the pupils and especially for pupils with learning difficulties or language difficulties.

A culture of rewriting and improving multiple drafts creates a significant change in the way in which pupils address their work and even themselves. The next stage in this change is that the pupils will give feedback to one another on their work, instead of all the projects being submitted to the teacher, for her exclusive judgment.

In project-based learning, time is dedicated especially to this feedback. This is an opportunity for every pupil to learn from the work of his classmates and to receive feedback for his work in a structured and guided manner, which does not threaten and is not upsetting. In the providing of the feedback it is important to be sensitive, considerate, precise, relevant, and in parallel to make suggestions for improvement.

Six Principles of Project-Based Learning. The educator Steinberg (in Patton, 2012) formulated a set of six principles for the planning of project-based learning: authenticity, academic rigor, applied learning, active exploration, adult relations, and assessment that includes many opportunities for reflection, both for teachers and for pupils, for holding at the end a summative event, such as an exhibition, show, or presentation, and for meeting standards accepted in the real world and in the school.

Advantages of the Method from the World Researches. In Maslow’s Hierarchy of Needs (Maslow, 1954), human needs contain five levels as a pyramid:

physiological needs, safety needs, needs for love and belonging, need for self-esteem, and self-actualization needs, with the largest and most fundamental levels of needs at the bottom, and the need for self-actualization at the top.

According to Xiaogin (2016), PBL motivates children and is effective in the improvement of the learning since it fills the three needs (from Maslow's theory) of adult learners: belonging, self-esteem, and self-actualization. Xiaogin (2016) examined pupils of business English and saw the effectiveness in their learning through projects, the response to the three needs of Maslow, their motivation to learn, and the long-term general objective of the improvement of the communication abilities.

In the aspect of the need for love and belonging, the pupils said sentences like being a member of a class or a project team, receiving love, affection, trust or care, from the teachers or peer learners, getting well along with team members cooperating with team members without conflicts, and gaining support or help at working on the project. In the aspect of esteem needs, the pupils said gaining the respect of team members or teacher, gaining self-esteem and self-respect, being competent for achieving some tasks under the project, and gaining recognition for your contribution to the team working. In the aspect of self-actualization the pupils said sentences like finishing the project on time, doing the presentation in English, exhibiting the development of a variety of skills and knowledge, and sharing both positive and negative experiences with fellow classmates and teachers.

Marwan (2015) sought to examine the nature of teaching English with information communication technology (ICT) and PBL. He saw that the learning becomes interesting, structured, significant, and rewarding for the learners. The teachers create a social atmosphere among their pupils, easily follow up after their pupils, and encourage activities of learning with peers through questions and corrections, and the obtained results is more effective learning. They prove good understanding of pedagogy and bring interesting materials and aids to teaching, and the learners assume responsibility and self-direction for their learning, investigate, and complete the tasks enthusiastically. They are active and responsible, they make decisions, and they are

motivated and confident in their learning. Because of the social atmosphere, it appears that the teachers and the learners are partners in the learning and in the making of decisions.

A research study conducted in a university in Malaysia in 2012 compared between two groups of pupils. One group studied in the conventional method, while the other group studied using project-based learning. A significant change was apparent in terms of the performances in the favor of the group that studied using project-based learning. This change was made possible because of the characteristics of project-based learning, such as cooperative learning, learning in small groups, discussions in groups, the presence of the teacher for help in learning, and resources for problem solving (Tarmizi & Bayat, 2012).

Another qualitative research conducted in South Africa examined during a year the project-based learning approach. The findings indicated that the exposure of the pupils to teaching in this way advanced in them significant patterns of learning that are characterized by the processing of topic critically and by processes that aim at independent learning and self-regulation over time. They used planning, problem analysis, product testing, adjustment and reflection of solutions, and metacognition. It seems that the pupils were less dependent on the lecturer. However, it was clear that there was tension on the matter of the change to the new approach. Some pupils insisted on obtaining correct and precise answers in the lecture and not through their experience (Malan, Ndlovu, & Engelbrecht, 2014).

In Canada a model was constructed for the increase of skills of problem solving for pupils of the undergraduate degree in education that included four components: (1) analysis of the preparedness, assessment of needs, and analysis (2) process of process of project based learning with the use of methods of discussion and through social media, planning, and development, (3) assessment of learning achievements and problem solving skills (outputs), and (4) evaluation of the results between the evaluation and the feedback. The model was proven to support teachers in education of pupils who participate in their learning (Jewpanich & Piriyasurawong, 2015).

According to Bridgeland et al. (2006), many high school pupils drop out because of the boredom and lack of relevance in the school. The school principals, on their part, attempt to reduce dropping out and to increase seriousness in the classes also through PBL, which has seen a revival in the past decade. Educational districts in the United States examine the PBL strategies to increase the relevance and to shift to requirements of basic standards to examine the pupils according to their products and not according to the learning by rote for the test. PBL entails the active and collaborative integration of pupils and places them in a realistic environment of real life experiences and problem solving. The activities were intended to promote a deeper level of understanding of the content significant to the learner, through cooperation.

Although the method is gaining momentum, the literature shows slow movement since Americans have fixed perceptions how to the school, especially the elementary school, should be, relying on traditional management. The teachers and the principals are challenged to find ways to improve the academic achievements, to interest the pupils, and to train them for the real world.

The implementation of PBL deviates from the traditional work in the school and challenges the practice of learning by rote as the sole strategy. The educational district in the present research study used the method as a new strategy for the improvement of academic achievements and training of the pupils in this era of responsibility and reforms based on standards. The implementation of the program has lasted two years in a school in South Texas. The research shows that the pupils' participation in the program positively influences the academic achievements in grades seven and eight in reading and in arithmetic (Cervantes, Hemmer, & Kouzekanani, 2015).

Teachers in a research study of Braran and Maskan (2009) received an opportunity to practice the method and found it to be beneficial. Their conclusion was that the programs for the training of teachers will give pupils opportunities to experience the method and practice it.

The teachers argued that PBL would develop abilities for the performance of the research, work in a group, and productivity (Larmer & Mergendoller, 2010). PBL was

found to help pupils develops skills appropriate for the 21st century, such as research, search for information, cooperation, communication, critical thinking, and use of technology. However, the difficulties the teachers indicated in the implementation of the program were the pupils' lack of interest, limited time, and problems in class management. The pupils could not analyze the workload in the project, wasted time, and left everything to the last minute. Hence, the pupils could not acquire the required skills.

The teachers maintained that they find it difficult to implement the method when they are not experienced in the planning of the stages but they believe that they will learn to do so and that the differences in the learning styles of the different pupils needs to be taken into account. This invites thoughts about the quality and effectiveness of the training of teachers in the field of PBL (Baysura, Altun, & Yucel-Toy, 2016).

Pinto (2013) in her article explained that pupils find it difficult to make the connection between mathematics and everyday life. She argued that when the project-based learning approach is adopted the pupils learn that geometry is not only theoretical but also practical and essential and beyond the basic understanding of the content they will enjoy the discovery.

III. Students of Teaching Mathematics in the Colleges

In this chapter, I will describe the students who learn the teaching of mathematics, the difficulties with which they cope and the diverse ways of learning through video, games, projects, online lessons, and special programs in the framework of the practicum in the school.

1. Students of Teaching Mathematics

The changing reality in the 21st century, which is characterized by frequent changes in the professional career, obligates re-thinking in the field of the training of teaching. The integrative model, in which studies in the field of knowledge and studies of training for teaching are learned simultaneously, exists in the academic colleges for education for many years, during which changes and developments occurred, both in the scope of the studies and in the learned contents. Some support this model of training, and some maintain that the linear model is preferable, in which first the area of knowledge is learned, an academic degree is obtained, and then the studies for the training for teaching are begun. At the end of the training a teaching certificate is obtained. An example is the re-training programs for people with academic educations or a master degree in teaching in the M.Teach program. The question of the preferred model has no unequivocal answer, not even in the research studies that examine this question, although in all models of training the component of the clinical experience is of central importance: practical experience and controlled teaching, the support of the pedagogical instructor, mentor teacher, peer teacher, and so on. Support of the importance of the clinical experience during the studies of the profession, before the entry into independent work, can also be found in additional professions, such as medicine, nursing, physiotherapy, and so on (Greenfeld, 2015).

In most research studies, **teacher training** was conceptualized as a "rational interest of policy" (Cochran-Smith, 2004), with the goal of determining which of the broad parameters of teacher training supervised by the shapers of policy will promote quality teaching and positively influence the desired products of the school. The research

studies adopt a 'rational' look at the setting and implementation of policy as a linear process guided by above (Datnow & Park, 2009), when the focus is the choices between alternative means for the achievements of regular/known goals found to be with broad consensus.

Although the research studies were clear in this conceptualization, it is possible to find those who engaged in the search for more effective ways in recruitment and training so as to solve important problems such as unequal distribution of quality teachers among schools. The goal is to provide the shapers of policy with knowledge for the educational reform in consistent ways for the neoliberal agenda, according to which accessibility refers to access to quality teachers. The assumption is that when people have access, many of the problems of school inequality are solved, since teachers and schools and not social factors are what determine products that can be measured by tests. Most of the research studies in this direction or in this research program do not raise questions on the structures and social systems that caused problems of inequality of this type (Cochran-Smith, 2016).

The subject of mathematics is a complex subject that requires of the teachers who teach it already in the elementary school not only mathematical knowledge but also content-pedagogical knowledge, knowledge on ways of thinking of pupils, ability to make complex material simple, and so on (Lindquist, 1989; National Council of Teachers of Mathematics, 1991; Shulman, 1986).

The good understanding of this subject from the beginning, from the classes of the elementary school, constitutes a good basis for the construction of understanding in the continuation. Therefore, the teachers in the elementary school, who are responsible for the way in which they will build with pupils the pupils' mathematical knowledge at the beginning of their path, needs to be an expert teacher in the teaching of the subject of mathematics. This trend began only in the past two decades in the elementary schools, since until then the requirement for an expert and professional teacher in mathematics was only in the secondary schools while in the elementary schools this was only a

recommendation (Lindquist, 1989; Mcnerney, 1994; National Council of Teachers of Mathematics, 1991; U.S. Department of Education, 1984).

In addition to the mathematical content knowledge and pedagogical content knowledge, there is agreement that the teachers in elementary schools need to have some concept of the ways of mathematical thinking of children. They need to be aware of the causes of widespread mistakes and be capable of understanding original ideas of children on 'doing math', whether these ideas are correct or not (Weiner, 2014).

Pecker (2016) found a negative influence of the dimension of mathematical content knowledge on the teaching effectiveness, motivation, taking responsibility, and self-orientation in the teaching of mathematics among students who are being trained for the teaching of mathematics. Similarly, the dimension of self-confidence and level of anxiety about the teaching of mathematics negatively influenced the effectiveness in teaching and self-orientation in teaching.

The programs for the training of teachers constitute the initial and basic opportunity for students of teaching to think like a teacher and learn to teach from a theoretical and practical perspective. However, all these programs do not prepare them for the true complexity of the professional work in the field (Greene & Campbell, 1993; Veenman, 1984).

The beginning teachers find it very difficult in the initial period of teaching since they must immediately implement the knowledge they acquired and develop skills and suitable positions (Flores, 2006; Hebert & Worthy, 2001). In addition, the burden of the responsibility limits their ability to learn how to teach. Therefore, it is important that the training and guidance will be sufficient while they are still students, so as to minimize the anticipated difficulties and to increase to the greatest possible extent the opportunities for the learning of teaching (Huling-Austin, 1992).

2. Training of Academically Educated People to Teach Mathematics

Academically educated people who are interested in any stage of their life in making a change and shifting to the teaching profession are an excellent resource for the educational system and they must be equipped with the same mathematical knowledge that the teachers need. Their relative advantage is not that they already know mathematics for teaching but rather their proven ability to learn mathematics and to learn in general. Obviously there are certain things they have already learned, and therefore their path to the mathematical knowledge may be shorter. Barbash (2016) disputes the prevailing assumption that they will already discover alone what knowledge they are lacking and will complete it in the continuation through in-service training workshops or by themselves. She asserts that this is only partially true, only on the level of 'hand-to-mouth'. They certainly will discover and learn by themselves the chapters of knowledge necessary for the coming lesson, but it is doubtful whether they will discover a lack in the more profound topics. It is true that if they feel that they are missing a formula, a theorem, or even a topic of teaching, then it is reasonable to assume that they will succeed in learning it by themselves. However, considerable time will pass and many lessons may be damaged or deficient until the teacher discovers – if she does – that she does not know the mathematics at the basis of the material she teaches.

In her opinion, it is not fair towards people who turn to teaching as a second career or as a mission and it is not right from the perspective of the system to put them into the teaching of mathematics without investment in their mathematics studies. She proposes to think how it is possible to shorten and ease the period of training for them, taking into consideration their academic past. However, they should not be placed into the classes without having learned the knowledge domain they are supposed to teach. In her opinion, it would be good if it were possible if for at least one year they would learn an orderly program of courses in mathematics and in mathematics teaching in parallel to the broad experience in the school and would receive a living stipend that would support them in this stage of their life in which they are turning to the educational system and then the continuation of their studies they could combine with the beginning of work as teachers. The financial investment would pay off (Barbash, 2016).

3. Difficulties in Mathematics with Which Teachers and Students of Teaching Mathematics Cope

Kurukkan (2015) believes that students have difficulties with mathematics since they do not remember what was learned in previous lessons, do not understand the concepts appropriately, and their teachers advance in the material rapidly without referring to them because of the large number of students in the class. However, it appears that children who have difficulties in mathematics act without motivation and without effort, refrain from asking for help, adopt superficial learning strategies such as repetition of what was done in the class and learning by rite, without new attempts. In this way they have less control over their learning, the difficulties become continuous, and the achievements decline.

Kurukkan (2015) maintains that students attribute failure or delay in mathematics to an internal reason, which is stable and outside of their control, when in actuality this is a controllable reason. His research indicates that the important step in the promotion of the students' efforts is to increase the interest in learning mathematics and to increase their self-efficacy.

Carpenter and Lehrer (1999) explain the rapid forgetting of the learned material in the absence of the deep processing of the material or the mistaken coding. If the strategy is only learning by rote, then the students with difficulties do not have the ability to connect a new question to what was learned previously. In their opinion, deeper learning strategies promote deep processing and thus long-term memory, and therefore to succeed in mathematics the students need to create rich and integrated knowledge. Sweller (1994) adds that a student who lacks supporting schema for the solving of problems will feel a high cognitive burden and will find it difficult to solve problems.

Some report that the student's progress depends largely on the teacher's quality (Mcber, 2000). Haimowitz (1989) notes that inadequate or inappropriate teaching is the cause of most failures in school. However, he focuses the point and states that a teacher can make mathematics easy only for those that feel it is an easy subject.

Mendelson (2013) indicates that the professional credo of the mathematics teaching trainees is linked with the promotion of the pupils' achievements and the raising of their motivation in the mathematics lessons and expresses a perception according to which it is necessary to see the place of the single student in the constellation and to focus on the value-oriented aspects related to him as an individual.

4. Inner Pictures of Teaching

How Do the Trainee Teachers of Mathematics Perceive Their Role?

Mendelson (2013) examined the way in which the trainee teachers for the subject of mathematics in the elementary schools perceive the images and the beliefs that shape this image through a series of seven metaphorical pictures. The picture of the 'animal keeper' was chosen by most of the participants. A total of 35% of all the participants chose it. Those who chose this figure as representing their professional image as teachers for the most part addressed the traits that characterize in their opinion the 'caregiving' teacher, such as the display of concern and care for the pupils and the great ability to listen and give. From their perspective, the mathematics teacher must teach the material in an interesting manner, but first and foremost she must support the pupils and encourage them also when they find it difficult to learn the material and fail in it. The picture of the 'conductor' was chosen by 33.3% of the research participants, and it is ranked in the second place of the 'animal keeper'. The students training for the teaching of mathematics described the 'conductor' as one who conveys confidence in her ability to lead her pupils to success, displays good mastery of class management, and is careful about reciprocal relations and holds an ongoing dialogue with her pupils. Most of the participants see positive interaction between the teacher and her pupils to be an important condition of the pupils' success and believe that only "seeing each and every instrument in his uniqueness may comprise the whole …"

From the seven pictures the pictures of the 'trainer', 'operator', and 'judge' were rejected by majority. The participants' explanations indicate that the every comparison of the teacher's image to one who displays mastery, forcefulness, and use of excessive authority towards the pupils inspires in them anger. In contrast, the picture of the

‘entertainer’ primarily awakened insult among the pupils from the non-Jewish sector, because of the possibility of seeing the teacher as an entertainer who is supposed to amuse the pupils instead of teaching them seriously, while the explanation for the rejection of the image of the ‘seller’ was linked to the fact that the participants refuse to see their role as selling merchandise.

The picture of the ‘animal keeper’ and the ‘conductor’ received few rejections. One student raised the fear that the care behavior may create an impression of loss of ‘distance’ between the teacher and the pupils, while participations who rejected the ‘conductor’ described the figure as exerting complete control over the orchestra (the pupils) and emphasized primarily his great distance from the pupils.

The dominant choice of the figure of the ‘animal keeper’, as well as the sweeping rejection of the figures of the ‘trainer’ and the ‘judge’, indicates the centrality of the topic of concern and caring in the training of the teachers in the college.

5. External Pictures of Teaching: Watching Video Films

Reflection is considered a main component in the teaching of mathematics in particular and in teaching and learning in general (Loughran, 2002). However, it is not possible to assume that the trainees know its meaning, since the process is hidden from them as learners and it is hard for them to analyze class events to identify the differences between the pupils and to support the learning.

Because of this need, researchers in Ankara created a different learning environment for mathematics teaching trainees, in which they watched and discussed video cases of real mathematics classrooms in an online discussion forum. The findings indicated that the future teachers who watched the video films noticed important issues related to their work as teachers and knew to reflect many areas and their ability to evaluate rose as they practiced the watching of filmed lessons more. The conclusion of the researchers from this research study is that the use of case-based pedagogy in the training of teachers is an effective way of helping the teaching trainees plan for the profession (Osmangolu, Isiksal, & Koc, 2015).

Stockero (2008) maintains that a curriculum based methodically on the documentation of the teaching events on video is a tool for the creation of the involvement of the trainees in the ongoing and focused reflection, as well as the transfer of learned abilities to action. The research respondents' ability to go from reflection on others to similar reflection on themselves is an encouraging finding for the teachers of teachers who search for ways to help cultivate reflective thinking.

6. Discursive Way of Collecting the Teaching Patterns - Online Courses

Research studies indicate that in principle the attrition of learners on online courses is higher than that of learners of traditional face-to-face courses (Smith, Ferguson, & Caris, 2002; Thompson, 1997). In essence, the dropping out from online courses depends on the learner's level of maturity, self-discipline, and adjustment to the online interaction in the online course (Roblyer, 1999). However, there is proof that experienced and dynamic instructors or lecturers can reduce the rate of attrition in an online course (Carr, 2000).

Smith and Ferguson (2005) examined the attrition rate of learners in online courses in mathematics in comparison to other areas of knowledge and in relation to traditional courses and discovered that unequivocally the dropping out from the online courses in mathematics is higher than in face-to-face mathematics lessons. They indicated two main causes, one related to the learners' background and the other, more important, related to the online means of the transfer and the method of online transfer.

First, the learners in mathematics online courses are generally older learners. They have greater gaps in the mathematical knowledge, since mathematics is a subject of knowledge that is steadily being accumulated and built, step by step, while in other areas of knowledge are less sensitive to systematically accumulated knowledge.

Second, the online learning environments used to convey mathematical knowledge are asynchronous environments that include in many cases technological means, such as discussion groups or email, means that cultivate more textual learning. These are means of learning that are not comfortable for the teaching of mathematics and

it is difficult to convey through them representations in mathematics (Smith, Ferguson, & Caris, 2002). Moreover, these are technological means that have a delay in time in terms of question and response (Smith & Ferguson, 2004). The existing pedagogical model of the holding of mathematics lessons in an asynchronous manner does not suit the teaching of mathematics at all. Synchronous communication does not especially benefit the holding of lessons in mathematics since the response time loop is too slow for learners who have difficulty in mathematics. In the learning of mathematics too much response time is necessary, for feedback and explanation that need to be given immediately without a delay (Smith & Ferguson, 2004). Learning in a computerized forum is not built on rapid response time but on the divergence of discussions. In mathematics it is more important to solve problems and conceptualize the way of the solution of problems than to hold discussions on a certain issue (Lester, Masingila, Mau, Lambdin, dos Santon, & Raymond, 1994). In comparison to online courses conveyed asynchronously, in face-to-face courses the feedback of the teachers is more dynamic. The teachers respond on the basis of the immediate feedback of the learner and can immediately adjust, broaden, or conceptualize the way of the solution (Smith & Ferguson, 2005).

7. Building Personal Patterns of Teaching - Training and Going into the Field

Brown and Borko (1992) deliberated on the emphasis in the training of students for teaching mathematics. They claim that without contemporary mathematics knowledge teaching students will devote considerable time to the learning of the material itself instead of planning how to present to the pupils. Perhaps it is necessary to focus on the increase of the pedagogical content knowledge of the students, since as known the pedagogical content knowledge is unique to the subject of mathematics and is expected to be insufficiently developed among young teachers and therefore needs to be the primary emphasis in the training program. Perhaps it is necessary to dedicate considerable time of the training to the experience in the field, since this is ‘the real thing’ and if young teachers receive too little experience or receive conflicting messages from the college lecturers and coach teachers in the school, their experience may not develop in the desired way (Vacc & Bright, 1994).

After the stage of the training, which is the first in the teacher's professional development, there is the second stage in the first years of teaching. In these years the teacher copes with many difficulties; some are unavoidable difficulties of a person at the beginning of his professional path, some are difficulties that derive from the manner of the training, and some are difficulties that derive from the lack of fit between the way in which the school works and what is taught in the raining. Papick et al. (1999) made changes in the training program in Missouri University following a joint project with teachers in the field who were at the beginning of their professional path, after they found that there are teachers who teach mathematics in the elementary school after partial training or without any training in this field.

The research participants were the young teachers, who told that the relevant courses in the period of the training for the teachers who teach mathematics are the didactic courses, the courses related to didactics and practical work. In their opinion, the mathematical courses were less relevant, while the general courses not related to mathematics were even less relevant.

They maintained that pedagogical instructors not from the field of mathematics accompanied them and that the number of lessons of observation and number of lessons of experience were insufficient. In addition, the coach teachers were not experts and professionals in mathematics. Most of the students saw the college to be responsible for their training but argued that it was necessary to make a significant change. Thus it was decided to re-examine the mathematical contents that the mathematics teacher in the elementary school must know, as well as the didactic components and the correct balance between the teacher's training as a professional teacher of mathematics and her training as a general teacher. In addition, it was decided to re-examine the way in which the practical work is undertaken and to enable the optimal conditions for the student, despite the more than a few technical difficulties that the college faces on this matter.

It was also decided to form shared criteria for training, through the examination of the possibility for a mandatory and joint curriculum for the different colleges, both for the students specializing in mathematics and for the students who are not specializing, to

write the teaching and learning materials for the different courses, which will help in the formation of the standards for the training of teachers of mathematics and to examine the role of the colleges as supporting the students they trained at the beginning of their path and as concerned about their professional development (Markovitz, 2004).

Schmidt, Bloemeke, and Tatto (2011) note additional factors that are entailed already in the acceptance to the trainings and the arrival at the school. They maintain that there are three interrelated structures that should be taken into account in the context of the training of the teachers for the future. The first is the quality of the students who are recruited to the teaching profession: in the countries in which the teachers for the future are recruited among those who excel among those with achievements in mathematics (Taiwan, South Korea, and Germany) teachers of higher quality are accepted. Second, the social and economic status of the teaching profession has impact on the recruitment: in countries where teaching is perceived as an admired and esteemed profession there are teachers with higher quality. Third, the organization and structure of the school system can promote or delay the quality of the teaching of mathematics (Schmidt, Bloemeke, & Tatto, 2011).

The integration of mathematical contents in the school teaching obligates 'didactic' processing or what Borovik (2012) called 'didactic transformation'. Didactic processing includes the simplification and adjustment of mathematical contents to the teaching needs in the school. The adjustment to the teaching needs on some level must be done so that the mathematical content will not be distorted during the didactic processing; otherwise an object or mathematical topic may become another objective to topic or completely lost any meaning.

Such processing of mathematical content is not a simple task, since it obligates a combination of mathematical considerations and pedagogical-didactic considerations. In addition, it is necessary to understand the need for learning this or another chapter and understand the role and position in the development of the pupils' mathematical knowledge over the years of learning (Barbash, 2016).

Borovik (2012) maintains that the transformation of mathematical content to the topic of teaching in the school is a mathematical activity. “We have to accept that in mathematics didactic transformation is indeed a form of mathematical practice. Moreover, it is in a sense applied research since it is aimed at a specific application of mathematics: teaching”.

Barbash (2016) adds that the simplification of a mathematical object to a level that enables it to be taught in the school creates tension between the ‘mathematical nature’ of the object and its representation in the curriculum and study materials. This tension may become unreasonable and intolerable if the ‘mathematical truth’ is brought closer to the didactic convenience.

“Unfortunately, the principles of didactic transformation are frequently neglected in the mainstream mass mathematics instruction. Hans Freudenthal even coined the expression 'anti-didactic inversion' … to describe the regrettable situation when purely procedural aspects of mathematics dominate the teaching”. (Borovik, 2012)

Barbash (2016) summarizes in her article that it is important that mathematics teachers be aware of the relation between the mathematical topic as revealed to the students in the different stages of learning and the ‘mathematical truth’, if our intention is for them to be the mathematics teachers and not teacher for some test or another, no matter how important.

Fares (2016) discusses the question of the positioning of the training as a topic for discussion for many years. Does the training need to be within the walls of the academic institutions or in the field, namely in the school, or in a balanced and adjusted combination between them? He holds that in his experience as a school principal it is necessary to allow parts of the training to be performed in the school field and to change the paradigm of the direction of theory-practice in the direction of practice-theory, of course in the proper ‘dosage’. He further adds that the teaching of mathematics involves personality and social components, and therefore he believes that it is necessary to develop new models to teach this subject, models based on the encounter with the field, including the principals, leading teachers, and students (Fares, 2016).

8. Mechanisms of Internalization of Teaching Patterns

A steadily increasing number of educational programs in the United States attempt to combine high thinking skills in the transference of the contents in the schools and therefore programs for the training of teachers there are required to adjust themselves to this. Sezer (2008) found that the combination of tasks for the development of critical thinking in the contents of the course for the training of teachers in mathematics had a great positive impact in comparison to courses that were devoid of thinking tasks. The students in the group who participated in such a course displayed more positive positions regarding the ways of teaching for the development of thinking and greater willingness to create challenging teaching in the class, in comparison to the positions of the students in the control group who did not participate in such a course that cultivates critical thinking (Sezer, 2008).

Today the importance of the extensive mathematical knowledge is steadily becoming more prevalent for teachers of the elementary school and therefore there is increasing need for in-depth mathematical courses in the training of teachers for mathematics. Students report a large gap between their learning in training and the traditional teaching in the classes. In actuality they attempt to emphasize problem solving, understanding, dialogue, use of relevant tools, justification, and reference to manipulations. These increase their confidence in their mathematical abilities. When they research the children's thinking, they encourage relevance, motivation, learning, and change, and these help them assess their future role as teachers of mathematics (Auslander, Smith, Smith, & Hart, 2016).

According to the research studies performed in education for engineering, the different ways that the mathematics teacher use to cause authenticity in the classes can be summed up in a framework that addresses the content, task, and influence. The authenticity can provide the diversification required for the students, so that the mathematics will not remain as a series of separate ideas that should be remembered but will become a cohesive body of personal, interesting, and beneficial knowledge (Garrett et al., 2016).

Pehkonen (1993) maintains that the instructors of teachers and students see mathematical problem solving as a way for the development of the cognitive skills of the pupils and help in the mastery of the learned mathematical material. However, knowledge about the history of mathematics and computer based instruments and resources in mathematics were not considered by the teacher instructors as important to the mathematics teacher. Some of the teachers thought that the history of mathematics in a certain way plays a neutral part in the teaching of mathematics. In their opinion, a teacher needs to know very little on the topic. The teachers who saw history as important to the mathematics teachers explained that it can serve as an analogy of learning or another aspect that requires the correct understanding of mathematics. Ernest (1993) adds that the teacher's perception of mathematics can create a basis for the mental models in the teaching and learning of mathematics.

One of the important goals of teaching students is that students need to obtain opportunities to observe, discuss, and practice the teaching of good mathematics. If the instructors do not update the knowledge about themselves, then the instruction will connect inappropriately to the theories of education and the student can think that these theories are not relevant. The results of Asikainen et al (2013) show that instructors needs to participate in in-service training about the knowledge of teachers in mathematics. Although there are strong relationships between the teacher's mathematical knowledge and the quality of instruction in mathematics, the relationship is composed of a number of factors. The same is true apparently also for the relationship between the perceptions of the instructors for teaching regarding the teacher's knowledge and the teaching of mathematics in actuality and the practices of instruction of the pupils, which is a topic that deserves more attention in the field.

Finally, Gazit (2013) in his research study showed how students of teaching mathematics have a positive attitude towards the integration of humor in the mathematics lessons. They see humor to be a means of the improvement of different components of teaching. According to their outlook, humor does not result in negative behaviors (Gazit, 2013). The participants' positions correspond with the recommendations of the research

literature, which emphasize the role of humor as reducing burnout, improving the self-image, and providing added value to the teaching process (McMahon, 1999).

9. Facing the Future – The 'Academia-Classroom' Program

Research studies that examined the training of teachers clearly show that it needs to be connected to the field and must provide the teaching trainees with the possibility of integrating between theory and practice. This integration obligates a close cooperation between the institutions that train for teaching and the schools and kindergartens where the graduates of the programs will work. Such cooperation can be found in the different types of partnership programs, in which training institutions are linked with schools and created communities of learners for professional development of all the participants in this association – trainers of the teachers in the academic institution, school teachers and students of teaching. This new system of relationships enables the inquiry and treatment of authentic problems that arise in the clinical experience, shared planning of the learning topics, experience in new ways of teaching, such as co-teaching of an experienced teacher and a teaching student, collaborations for action research, and so on. The research emphasizes that to promote this trend an essential change in the perception of the structure of the training for teaching is necessary. In the traditional perception, the training for teaching is built from an orderly system of academic courses, which were supported by clinical experience, whose role was to clarify and illustrate what is learned in the academic course. In the new approach it is suggested to build the training for teaching as a meticulously designed system of clinical experiences, supported by theoretical courses that have the goal of the development of knowledge, skills, and positions of the trainees for teaching, which will be required of them so as to promote their pupils in the different schools where they will work.

One of the programs that have been successfully implemented in Israel is the 'Academia-Class' Program. This program combines students who are training for teaching as additional teachers in the classes where there are a large number of pupils, for two reasons. The first reason is to allow the student to have an authentic experience of teaching already during her studies alongside a skilled teacher. Thus, the student will be

more prepared for the challenges she faces with her entry into the classroom as an independent teacher. The second reason is to allow 'teaching in pairs' of the student-teacher and to dedicate greater 'teacher time' to every pupil.

The 'Academia-Classroom' Program causes the improvement of the training for teaching and increases the preparedness of the new teachers to enter the school as independent teachers. In this model of training the students' clinical experience is increased and they experience the school culture and are partners in it through the integration of academic knowledge and practical knowledge. The program contributes to the increase of the academic institutions' involvement in the schools, to the design of pedagogical instruction, and to professional development of all the school teachers. In the schools a path for the promotion of the training teachers will develop, when students will be assigned to them as a part of their professional advancement. The program will contribute to the increase of the achievements of the pupils in the schools, who will receive greater attention and answer to their needs. The program will boosts the involvement of the districts, the local governments, and the centers of in-service training centers in the process of the absorption of new teachers and their integration in the educational system (Greenfeld, 2015).

IV. The Methodology of the Current Research

In this chapter, I will describe the research objective, research questions, research process, research population, research instruments, main ideas in qualitative research, the different methodological paradigms, and mainly the way that I chose, action research, in the constructivist approach that focuses on the research participants.

1. Conceptual Approaches to the Current Research

The methodology reflects the structure and components of the research study, and is in essence the operative way of the design of the research (Guba & Lincoln, 1998). The methodology is presented as a branch of philosophy that engages in the methods and rules for the performance of the research. According to the definition of Guba and Lincoln (1998, p. 201), methodology is the process in which "the researchers act so as to know what they think it is necessary to know".

In this work I review the characteristics of qualitative research according to different researchers and the methodological combinations in it. I note the different research paradigms: the positivist approach, the post-positivist approach, the critical approach, and the constructivist-interpretative approach. In addition, I present the six methodological patterns of qualitative research: the pattern focused on criteria, the pattern partially focused on criteria, the pattern focused on the respondents, the pattern focused on the self, the pattern focused on language structures, and the pattern focused on criticism.

I focus on the research type I chose, **action research**, on the **constructivist-interpretative approach** I adopt, on the **qualitative-methodological** pattern, which is **focused on the respondents**, on the research instruments unique to this pattern, and on standards of research quality. The methodology I describe includes the research objectives, and the research questions. According to the research type, **action research**, the **constructivist-interpretative** paradigm, and the methodological pattern which is **focused on the respondents**, I describe the population of research respondents, the use

of triangulation, the sample, the stages, the different research instruments, the rules of ethics, and the way of collecting and processing the data.

2. Research Objectives and Research Questions

The objective of the research is to describe the quality of the students' projects and products, reflection about their learning process, its socio-psychological environment, and their behaviors in the learning process in the context of teaching based on PBL.

Research Questions

1. What is the level of quality of the project and the products made by students in the learning process based on PBL?
2. What reflection do students have on the learning process based on PBL?
3. What kind of opinion do the students present about the socio-psychological environment they experienced during the learning based on PBL?
4. What kind of the student behaviors were observed during the learning process based on PBL?

Clarifications

The term **"**level of quality of the project and the products**"** includes the entire scholastic experience in terms of *authenticity and relevance activity, inquiring activity, applied learning, academic seriousness (meeting standards), relationship with experts, response to diversity, ways of assessment, and degree of autonomy granted to students*.

The term "the socio-psychological environment and kind of student behaviors engaged in learning by projects" means *aspects of curiosity, motivation, flow, self-orientation, self-regulation, feelings of satisfaction, self-efficacy, reflective-critical thinking, and belonging to the mission.*

Assuming that I will prove the effectiveness of project-based teaching\learning and the satisfaction of the learners who use this method, I can assimilate it among my

fellow lecturers in the Gordon Academic College, so that many students will learn the way and it will become common in schools.

3. Qualitative Research

Four Methodological Research Paradigms. Guba and Lincoln (1998) maintain that "both quantitative research methods and qualitative research methods may appear in a suitable way in every research paradigm. Questions of method are secondary to questions of paradigm…" On the basis of this assertion, they propose a distinction between four paradigms of qualitative research: the positivist approach, the post-positivist approach, the theoretical-critical approach, and the constructivist-interpretative approach. In each one of these approaches there may be qualitative and quantitative methods according to the research paradigm and the research goal.

The researchers who adopted the positivist quantitative paradigm guide themselves generally to use research methods that realize the ideal of formal mathematical methods and strive for universal generalization free of specific contexts. **The constructivist qualitative research paradigm, in contrast, guides its adopters in a completely different direction, based on narrative context-dependent contexts**. These two approaches are based, therefore, on completely different fundamental assumptions and it is impossible that research arguments that derived from one approach will be validated or refuted using research arguments based on the other research approach (Guba & Lincoln, 1989). Every qualitative research methodology includes both intuitive research abilities and analytical research abilities, both internal characteristics and external characteristics, and thus pure perspectives can never be talked about.

Denzin and Lincoln (2000) assert that the term 'qualitative research' has different meanings in different and changing contexts. Thus they propose a comprehensive definition of qualitative research. Qualitative research is "situational activity that proposes a point of view to look on the world … this means that qualitative researchers investigate things in their natural place, attempt to find meaning or interpret the phenomena in terms that people use."

Shkedi (2003) describes qualitative research using the following three characteristics.

1. It exists in the language of words and in the natural environment of people.
2. It is based on people's intuitive inquiry abilities and strives to closeness to the researched phenomenon, to involvement in it, and to empathy towards the research respondents.
3. It uses people's analytical inquiry abilities and strives for distance, reflection, and supervision of the research process.

The different methodological processes of qualitative research are distinct from one another according to the weight given to each one of the last two characteristics and according to the nature of the first characteristic.

Six Approaches in Qualitative Research. Shkedi (2011) speaks about six methodologies of qualitative research: methodology focused on the respondents, methodology focused on the criteria, methodology focused partially on the criteria, methodology focused on the criticism, methodology focused on the self, and methodology focused on the language structures.

The researchers in the *criticism-focused methodology* let their intuitive abilities 'read' the researched phenomenon, but support this impression with a critical analytical look, which sees the investigated reality with standards that go beyond intuition and impression.

Research studies in the *self-focused methodological pattern* engage in the 'self' of the researchers and not on the external research objects like other qualitative researches. Some researchers hold a theoretical perspective before the research, while for other researchers the theoretical perspective forms during the research.

Researchers who focus on *language structures* emphasize the understanding of the structure of the language and the social-cultural meaning of this structure. Research studies in structural methodology strive to reveal the relation between the aspect that

originates in the research of language structures and theoretical aspects that originate in the sciences of man and society.

Researchers who adopt the *respondent-focused methodology* will generally be those who identify with the constructivist-interpretative philosophy (Guba & Lincoln, 2005). The researchers do not rely in a binding manner on a regular theoretical perspective. The assumption is that this perspective becomes clear during the research (Creswell, 1998). They do not arrive at the research field with a crystallized theoretical perspective but they also do not come to the field as a blank slate (Pinker, 2002). The focus in this methodology is on the research subjects' experience and perspective. Each one of them constitutes a unique 'case' (Stake, 1995). The number of research respondents is limited relative to quantitative research studies.

The *criteria-focused methodology* is identified with the positivist paradigm. There is an external theory that guides the research study from its start to its end and it guides the shaping of the research questions and its hypotheses and the structured methods and the regular categories that are determined before going to the research field.

The *partially criteria-focused methodology* has an outside theory that guides the research from its start to its end but through discovery and flexibility. The research stages are prepared in a linear manner through the display of flexibility. The chosen methods are structured and have a certain degree of openness.

Sabar-Ben Yehoshua (2016) maintains that there is no doubt that the use of qualitative methodology entails the change of the paradigm, the adoption of a post-positivist worldview contradictory to the viewpoint of the quantitative-positivist paradigm, which still prevails and according to which many were educated. Every world picture is based, as known, on its own fundamental assumptions regarding the nature of reality and our ability to know it, on the relations between the researcher and the subject of his research, and on criteria for research trustworthiness. In the positivist paradigm, the assumptions regarding the nature of reality are different from the structural assumptions (constructivist) and are based on the belief in the existence of one absolute reality, with an external status.

This reality does not depend on time or context, and it is possible to reduce it to simple components among which there is a statistical relationship. Research aspires to discover this reality using instruments considered objective. In contrast, in the constructivist paradigm, reality is perceived as a human structure shaped by the research subject's cultural and personal conditions. This reality does not exist without a research respondent, since the respondent is a part of the reality. The reality is built from the interpretation undertaken both by the researcher and by the research respondent, and it is the basis for the structural perception (Sabar-Ben Yehoshua, 2016).

4. Methods of Data Collection in Proposed Research Methodology

The objective of qualitative research is therefore to discover the reality. According to the constructivist perception, it is impossible to achieve this goal, since there is no one absolute and correct reality. Rather, it is necessary to research different interpretations of the same reality through the construction of a clear experiential memory, which will help us describe and explain things with their multiplicity of facets and complexity.

Qualitative researchers use a variety of ways to collect information while cross-checking viewpoints (triangulation), so as to increase the trustworthiness of the findings. This, for example, is through the combination of interviews, observations, photographs, films, documents, and objects. Each one of the ways of qualitative research reflects the world in another manner to a certain extent, and the use of a number of ways indicates the researcher's intention to have in his research study more than one interpretation. The variety enables the simultaneous presentation of a multiplicity of realities and not any objective reality.

The idea regarding many constructs of personal and cultural reality is the basis of the qualitative perception. The multiple viewpoints demand a holistic approach, one that does not have the possibility of removing single variables from the overall context.

Qualitative research entails the collection and scholarly use of a variety of empirical materials – case study, personal experience, internal observation, life story,

interview, objects, ceremonies, and cultural products alongside documenting, historical, and visual texts of interaction. All these can describe, for instance, a moments and meanings of routine and crisis in people's lives.

Accordingly, Sabar-Ben Yehoshua (2016, pp. 13-14) adds that qualitative researchers implement a wide range of interpretative practices related to one another from the constant hope to achieve a better understanding of the discussed topic. However, there is the understanding that every practice makes the world seen in a different way. Therefore, frequently there is commitment to use more than one interpretative practice in every research study.

Standards of Quality for Research. To confirm the validity, reliability, and generalization ability of the arguments, through triangulation, the researchers must create an open and disclosed collection of data, document and preserve as evidence all the processes of analysis given to their types and stages. Since this is a research study based on the language of words, the research quality will come from the writings and the clear presentation and not through the statistical formulae (Shkedi, 2011).

Triangulation. Triangulation of methods and information sources is expressed in the multiplicity and diversity of methods and may strengthen the validity of the research and encourage diversity in the research methods (Merriam, 1998; Stake, 2000).

Despite the considerable importance of the triangulation the researchers set for themselves boundaries and limits. Despite their desire to enrich the triangulation through the multiplicity of methods, it is necessary to adjust the methods to the type of research that the researchers are performed and thus prevent the 'combination of two things that don't go together'. (Sabar-Ben Yehoshua, 2016).

The collection, processing, and analysis of the data will be performed through triangulation in a qualitative analysis.

Triangulation of Research Instruments – Interviews, Observations, Documents. In the qualitative method the observations, interviews, and documents were

examined - draft in organization, reflections during the process and at its end, and summaries of group and plenum feedback.

5. Action Research

The research type chosen for this research study is an action research. Action research examines the influences of the intervention and includes firsthand investigation by the person who initiated the action and is involved in it (Cohen & Manion, 1989).

Kinney (2006) asserts that action research is addressed as a special case of qualitative research, in which the researcher's involvement is maximal. The goal of action research is the transition to the structuring of the knowledge and the enrichment of the scientific community in the researched field. In action research, the researcher researches his action by himself so as to bring about professional improvement for himself, for the researched context, and for systemic professional change (Carr & Kemmis, 1983).

Levin (1996, in Sabar-Ben Yehoshua, 2016) indicates **six principles of action research,** which support the constructivist approach and are realized primarily because of qualitative research: **collaboration, reflexivity, dialectics, risk, pluralism, and return**. I present these six principles as they are expressed in one of the models of action research I will adopt for this research.

The three models that formed are the technical-traditional model, the practical-cooperative model, and the critical-emancipatory model. Sabar-Ben Yehoshua (2016) argues that the division into models was supposed to enable the researchers to position their work in the conceptual framework of one of the models and to protect it in the epistemological context of this model. However, sharp distinctions do not yet exist between the research methodologies (Sabar-Ben Yehoshua, 2016). In this part **I focus on the critical-emancipatory model that** I adopted for my research.

Action Research in the Context of the Critical-Emancipatory Model. The theoretical background of critical-emancipatory model in the field of education is critical pedagogy in the educational thought of our time (Gur-Zeev, 1996, 1997).

Sellermeir (2016) maintains that the emancipatory model is based on the postmodernist perception of change. In other words, in her opinion research does not necessarily have to initiate and establish a process of change since during our personal and professional lives we experience a process of unending change with which we must learn to live. She states that we aspire to utilize the general process of personal and professional change during the social action. Now, more than in the middle of the 20th century, we seek to see the individual in society not as an object or a victim of processes of change, but as an agent of change of himself and significant others who act together in the same context.

According to the critical-emancipatory model, the quality of action research is determined according to six principles (Sellermeir, 2016):

1. Principle of Reflexivity. Reflexivity is expressed in the space that the research builds for the participating educators both to re-see the personal and professional story of their lives and in the political-social context of their work and to examine the norms of this relationship and the influence on their lives. It is important to note that in critical action research the research devices are supposed to be not only devices for the collection of data but also mechanisms for the implementation of the process of reflection. When teachers analyze data collected about them and conduct a discourse on their findings, they have the tools for reflective activity that may cause in them a transformative process with meaning to their professional lives. This is a critical process, in which they analyze events together and openly so as to transform them.
2. The Principle of Dialectics. As aforementioned, analysis of the discourse reveals the shared topics of the participants' stories and the contradictions and conflicts among them. During the analysis, there is expression of the participants' objections to the topics raised and the work method in the project.

3. Principle of Risk. Like in other models, here too the participants in the research put themselves at risk, not only since they do not know where the research will lead but also primarily because of the exposure of the mechanisms of repression that they retained as a defense against their involvement in the research. Moreover, as a part of the process of liberation, they must act in a subversive manner against the mechanisms of repression and refuse to cooperate with the existing social order. Thus, they expose themselves to the copings that may harm them.
4. Principle of Collaboration. In the framework of their collaboration, they are called to spontaneous activity and volunteering from their free will for the development of ideas and initiatives of their own, through the deviation from the limitations of time and place and the display of ability to respond to opportune needs that were not anticipated. She reinforces the idea that the researcher from the academia, who dedicates considerable time to work with the teachers, has the legitimacy to present his interests to the participating teachers and to ask them for help in the realization of his professional needs.
5. Principle of Pluralism. The collaboration in critical action research is not intended to achieve a situation of equality. Rather the reverse is true: collaboration is intended to enable each one of the participants to emphasize his difference and make his personal voice heard. Therefore, the narrative analysis method is accepted here, placing maximum weight on the uniqueness of every story and every interpretation, while intentionally ignoring the supposed objective truth or the drive to reach any generalization.
6. Principle of Return. The participants must show how the research process constituted a process of return for each one of them and how they built through it their abilities to serve as agents of change in the system (Sellermeir, 2016).

6. Research Instruments for Qualitative Analysis

Content of Documents

Qualitative analysis documented the process until its end, such as drafts of planning and work, reflection, and feedback in the PBL courses. In these documents the six elements of project-based learning were examined – authenticity, academic rigor, active exploration, applied learning, adult relations, and assessment. In addition, the response to difference, the degree of cooperativeness, the use of advanced technologies, the level of curiosity, the flow, the self-orientation, the enjoyment, the motivation, and the enthusiasm from the learning and the summative project were examined.

Semi-Structured Participative Observations

Qualitative analysis of films and documentation of the participative observation of me and of my students who were chosen randomly as observers in the PBL courses. I recognized a difficulty that lies in the fact that I was teaching the courses, and therefore the observations were participative and there was a chance of research bias without feeling it. Hence, I chose from among the students a few who functioned as teachers and a few who functioned as observers.

One of the group members served as the observer and another one served as the teacher who directed and led the group.

Semi-Structured Interview

The semi-structured interview was distributed at the end of the course during the semester vacation. Students who agreed to be interviewed were interviewed. I began with an open interview and as needed I used questions from the interview presented in the appendices. The questions from the semi-structured interview:

- What's your name? Tell about yourself.
- What is your experience in teaching? What is your educational credo?
- What is your opinion on project based learning?
- Describe your feelings following the course.

- What was difficult? What made you enthusiastic?
- What is the value of such learning?

In the first semester of the 2016-2017 year, in the beginning of November, I began the action research in two courses that I taught in the Gordon Academic College in Haifa. The courses were intended for students who are training for the teaching of mathematics, both regular students and academically educated students.

The course I initiated is called the Development of Initiatives and Projects in Mathematics. It engaged in project-based learning in mathematics. In each one of the courses there were about 45 students in each course altogether about ninety students, who were divided into groups according to their shared interest in mathematical skill and in the project.

7. Research Procedure Used in the Current Investigation

Stage 1: Three introductory lectures on PBL on topics such as definition of PBL, work process through a project, historical background, rationale, and hypotheses in relation to pedagogical effectiveness.

- Choice of a productive question, such as: What is mathematics in everyday life?
- Joint thinking in the plenum on mathematical skills and division of the students into groups according to their interest in the mathematical topics and in ideas for projects.
- Choice of two students from the group members. One served as a mediating and directing teacher and the other as an observer.

Stage 2: Short presentations in which the main ideas were presented by the group members (preparation of the group members by the teacher – the student). Every group presented another topic.

Stage 3: In the following six sessions, there was work on the project while writing multiple drafts, reflections, and feedbacks in the group and in the plenum, from the focus on the specific mathematical skill, learning on the teacher level, planning, setting time

schedule, inquiry, relation with experts, building, changes, corrections, formative and summative assessments, until the summative event.

Stage 4: In the last three lessons – presentations of the summative events and reflections in each one of the groups.

Stage 5: Semi-structured interview in the end of the course.

8. Research Group

The research population consisted of the students in two courses: in one course, there were 42 regular students who were training to teach mathematics in the elementary school (third year) and in the other course there were 45 students who were undertaking academic re-training for teaching mathematics in the elementary school. In both courses mathematics was learned through projects. The population consisted of Jewish, Druse, and Christian and Muslim Arab students.

9. Process of Analysis of the Findings.

According to the main strategy of analysis of grounded theory, the constant comparison, I use four principles: proximity, theoretical sensitivity, continuation and multiplicity, and reflexivity (Givton, 2001).

I coded the parts of the text into categories through the comparison of datum to datum and the identification of shared meaning and patterns. I placed the texts in proximity so as to create encounters between the texts themselves.

The categories that recurred in the reflections written in the interviews, in the drafts of the 'teachers' in the groups, and in the observations of the observers of the groups are: concerns, curiosity, motivation, authenticity, relevance, esteem, critical thinking, satisfaction and personal satisfaction, applied learning, flow, investigative activity, difficulties, relationship with experts, self-regulation, academic seriousness, collaboration, and belonging to the group.

In the continuation, I will anchor the findings with the theory and will analyze when in each stage the power of the interpretation and simplification will grow. Throughout the process I can continue, return, examine the product of analysis and improve it.

Additional data in the 'back and forth' reading, the meanings that arise, the circumstances, and ideas for conceptualization bring me back or advance me in the process.

The continuation of the analysis and the many interpretations are essential and important, with the need for flexibility, extension of the meaning, and preservation of both "the voice of the researcher and the voice of the research respondents". (Givton, 2001).

In a more advanced stage, the reflexivity is addressed. The qualitative researcher is required to be aware of the process of analysis (Anfara, Brown, & Mangione, 2002; Peshkin, 2000). She must clarify well the perspective from which she reads. In texts, what is her positioning versus the researched reality? What of the mindset that she has influences the analysis?

Josselson (2004) focuses on the difficulty clarifying what of the researcher's self and private worlds is relevant to the research and the interpretation. Moreover, she notes that it is difficult to know how the relevant aspects indeed influenced the researcher's interpretation to give an accounting for her decisions and analyses. Did she suspend and 'frame' her early outlooks? Did she use them and 'exploit' them so as to interpret the research? She seeks to pay special attention to the change that occurred in her. Are her understandings at the beginning of the research different from those at the end? (Givton, 2001)

Givton (2001) proposes five stages for analysis and I act according to them, as follows.

The First Stage: The identification and definition of primary categories and the beginning of the stage of the theoretical sampling. In this stage, the researcher goes over

the initial material of the findings at his disposal and identifies in it repetitions. The intention is themes, topics, and titles that recur and can be characterized or implemented. The researcher divides the material according to themes or topics according to the units of analysis he determined for himself.

The themes identified in the first stage will establish initial categories and will serve as guides for the continuation of the process of the construction of the research, what is called theoretical sampling, and it means that the data that we find and the analysis and their primary interpretation direct us to additional respondents with which it is possible to converse.

At this stage, it is important to define the unit of analysis, or in other words the smallest unit into which the material of the findings will be disassembled. This is the unit that will compose the categories that are steadily being built. The units of analysis can be diverse, ranging from the statement, the response, a wall drawing, or an interview that describes a complete respondent – all according to the question we ask.

The last action in this stage is what is called the noting down of cognitive memories. This refers to the researcher's comments that link among the findings, between one category and another, between the findings and the categories and the concepts from the field of the research literature on the topic.

The Second Stage: The shaping of categories, the definition of criteria for categories, and the continuation of the theoretical sampling. In this stage, there is a process of analysis on the axis of the categories that have already been phrased, with their constant improvement. In this stage, there is a more precise definition of the categories, when the category must explicitly indicate the unique contents that arose under this theme. In addition, every category will receive criteria when it is necessary to divide all the material into these categories, even if there is a need for a category of different items.

The Third Stage: The refinement of the criteria and shaping of the final system of categories. Now the analysis is nearly complete, and it includes a system of categories and criteria. In this stage the researcher accumulates additional data and codes them into

existing categories. It should be noted that in every stage it is possible to go back and to reshape the categories and the criteria. This stage ends when there are too many repetitions in the material collected and the research reaches saturation.

The Fourth Stage: The creation of hierarchies between the categories and the identification of the core categories. In this stage it is necessary to link among the categories in different ways. In addition, it is necessary to identify core categories – the main categories that are linked to many categories, to be the heart of the analysis and to appear at high frequency. In addition, it must connect to and address the other categories and have relationships with the discussed issue.

The Fifth Stage: The creation of a theoretical structure through categories, the research literature, and theories. In this stage, the grounded theory is built and written, according to categories, the research literature, and the previous theories in the field of research. A good theory has characteristics: it is a functional theory, or in other words, it succeeds in explaining to different communities that the behavior in the social behavior is associated with the core categories and it needs to be valid also in the eyes of the respondents.

Data Processing in This Research Study. After repeated reading of the observations, reflections, and transcriptions of the interviews, initial categories were formed for the classification of the data. First thematic analysis was performed for the reflections, observations, and interviews of the students from every course, so as to identify shared patterns for every group. In the next stage the analysis was based on the identification of the themes and the coding of the information (Shkedi, 2003, 2005). The data were processed in three stages: (1) open coding – in each one of the reflections the topics in which there is a relationship between them and the research topic were identified, (2) axial coding – the creation of topic generalizations (categories), and (3) selective coding – additional examination of the categories and secondary categories obtained and surrender of the categories that are not necessary (Seidman, 1991; Strauss, 1987).

According to the hermeneutic approach, the texts in the observations, interviews, and reflections were "disassembled" into small secondary units. Then thematic topics noted in all the observations, reflections, and interviews were identified, and from them the supra-categories were derived. A complete and new picture was built from the data and categories (Kassan & Krumer-Nevo, 2010).

In the next stage, the data that arose from the reflections were compared to those that arose from the interviews and observations. Main topics were identified and characterized to follow up after recurring patterns in the students' statements in general and in the group context in particular. In addition, the frequency of their appearance in the interviews of aspects not noted in reflections and in observations was examined, and the categories were "thickened" as necessary. Some of the statements moved from one category to another for greater accuracy.

10. Rationale and Objectives of the Course Studied: Development of Initiatives and Projects in Mathematics

We learn from the research studies that emotions such as fear, frustration, and disappointment are found to inhibit the learning of mathematics, while positive emotions improve the quality of the learning.

The continuous process of the teaching of mathematics can be diversified and enriched. It is possible to create interest through the integration of creative materials, mathematical initiatives, and management of mathematical projects (in interdisciplinary topics) that may constitute a challenge for the good students and awaken curiosity and interest among all the students. All these will cause positive emotions in the context of the engagement in mathematics and apparently will increase the willingness to cope with mathematical challenges.

This course inculcates the methods and tools required for the implementation of Project Based Learning (PBL) in the classroom. The participants will undergo the entire process of planning the project in teams, from the choice of the idea, formulation of the essential question, planning the presentation of materials, integration of elements from

the curriculum of the Ministry of Education and skills of the 21st century, and building an effective system of evaluation. In addition, the participants will acquire knowledge in the management of projects, including ways to initiate a project, creation of work teams, encouragement of the use of methods of research and problem solving, the recruitment of the students for significant learning and the achievement of a quality outcome at the end of the project (Hilai, Course Syllabus, 2017).

Objectives

- To know the model of PBL and the ways to successfully implement it. Understanding of the characteristics of PBL and the skills acquired during it.
- To acquire tools for the building and management of work teams.
- To instill the ability to build a project from the beginning to the end.
- To develop school initiatives in the view of mathematics as an interdisciplinary subject. To actively illustrate the preparation of initiatives and to perform them through their modification in the integration of additional areas of knowledge. Inquiry tasks and tasks from everyday life.
- To learn the method for planning a project that integrates the teaching of skills and the teaching of contents and that enables creativity on the part of the students.
- To integrate methodologies of problem solving and planning so as to increase the project innovation.
- To be familiar with the advantages of active learning in mathematics.
    - To cultivate significant learning in mathematics.
    - To cultivate an independent learner who is responsible for his studies – strategic learning of management of the different environments of the venture and the reciprocal relations between them.
    - To develop diverse ways of teaching that will provide a solution to the different learning styles in mathematics.
    - To use ways of alternative assessment – identification of the ability of persuasion and dissemination beyond the mathematical knowledge.
    - To create experience and enjoyment of the process of learning and to remove fears of mathematics.

  - To encourage the ability of invention, development of creative thinking, imagination, and originality.
  - To inculcate skills of marketing, persuasion, and dissemination.
- To understand the role of the teacher in the assimilation of the initiative – mediation, guidance, and assessment.

Mathematical Topics from the Curriculum for the Elementary School

- Number theory – Actions, arithmetic stories, and investigation of whole numbers and rational numbers, and geometry of space.

V. Findings - Project-Based Learning (PBL) as a Promising Challenge for Prospective Mathematics Teachers in Math in Elementary School Education

1. The Fourteen Products

The guidance given to the students for the work on the projects was open and vague, but obligating authenticity and relevance, since they were asked to become 'startup founders' and to prepare an innovative and original product that would be an important teaching aid for teachers of mathematics in the elementary school in any mathematical skill that they would choose. Indeed, the products constructed were for the most part original, unique, and meaningful to each mathematics teacher. The students felt tremendous pride in presenting them.

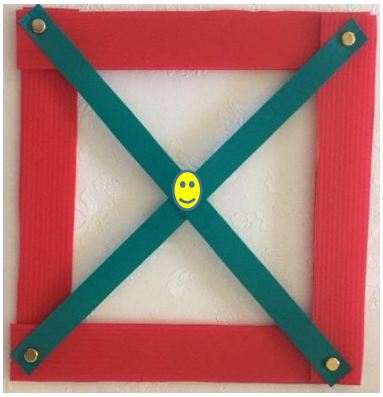
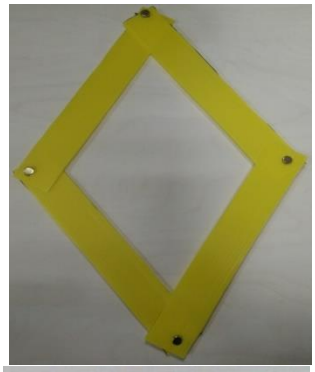
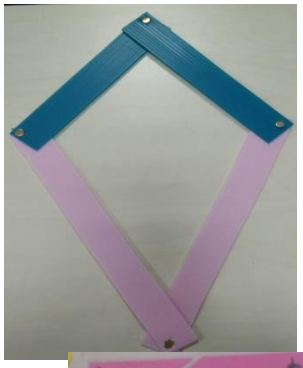
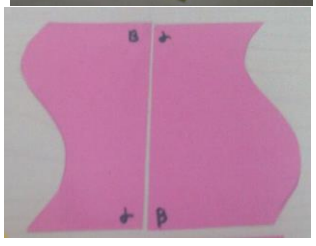
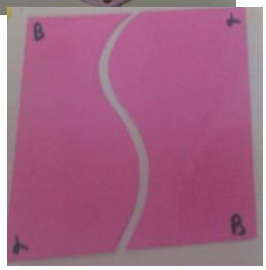
Project: Ratio Clips. The group that worked on the project on the mathematical topic of the ratio built a series of thirteen film clips. After learning all the mathematical skills on the topic of the ratio, reading articles on the topic, and my guidance, the seven students decided to build a series of clips that inculcate the basic skills on the ratio: ratio between quantities, ratio between quantity and whole, finding the ratio according to the given quantity, and division of quantity according to the given ratio. For every lesson in which the students worked independently they brought other items of illustration, such as fruit flavor concentrate with bottles of water, beads, candies, chocolates from a chocolate box, and stickers. They worked in an available classroom, wrote a script, directed, and filmed.

One student filmed each time, another student was the 'teacher', while another one was the role of the observer, and the rest were students. The films were directed as mathematics lessons with many illustrations and a mathematical conversation that is accompanied by appropriate concepts and diverse strategies. At the end of every film, a question was asked, so that every teacher in the class could use the films in the lessons, show them, stop the film periodically, and inspire a class discussion. In the continuation of the film, the answer appeared as given by the student actors.

Project: The <u>Family of Quadrilaterals</u>. The product of the projects includes a kit for fifth grade pupils with which it is possible to learn about the attributes of quadrilaterals and the relationships of inclusion between them. The kit includes teaching aids built by the students in the Pedagogical Center of the college using rigid polycarbonate sheets, dividing pins, and stickers. In addition, the kit includes a workbook with instructions, inquiry, and practice.

Examples of the teaching aids:



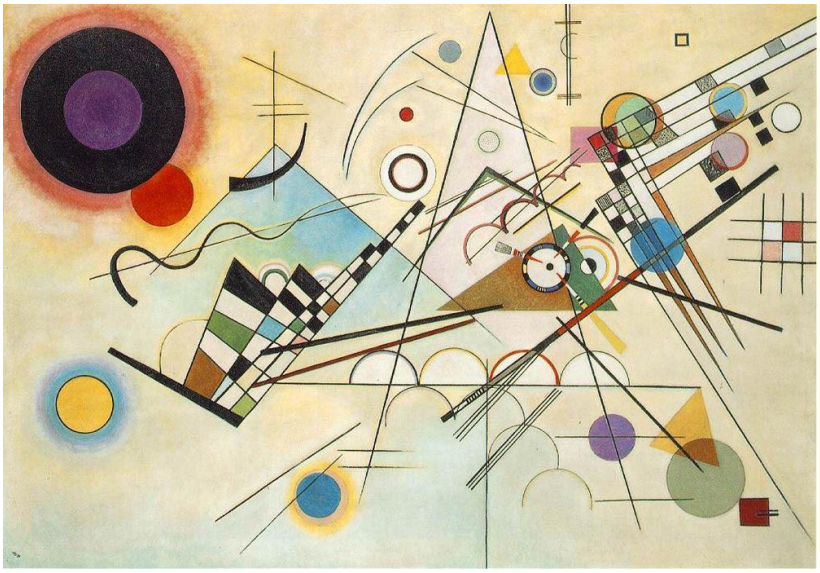






Wassily Kandinsky – 1866-1944

Kandinsky was a Russian artist and theoretician of art and is considered one of the famous artists of the 20th century and a trailblazer in abstract art. One of the components of his work was geometric shapes.

Look at the painting below and find:

1. How many quadrilaterals are in the paining? Find at least four.
2. How many squares are in the paining? Find at least four.



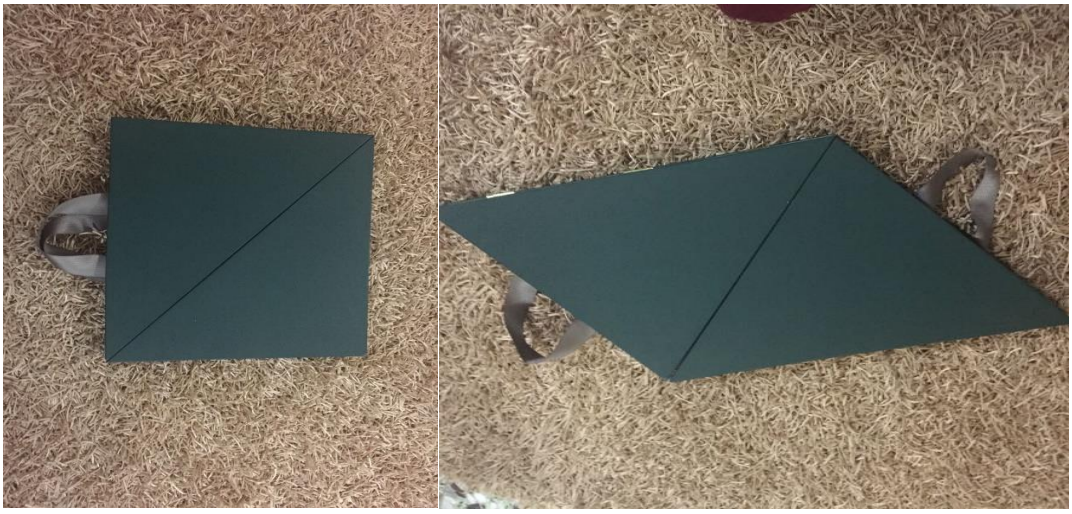
Parallelogram

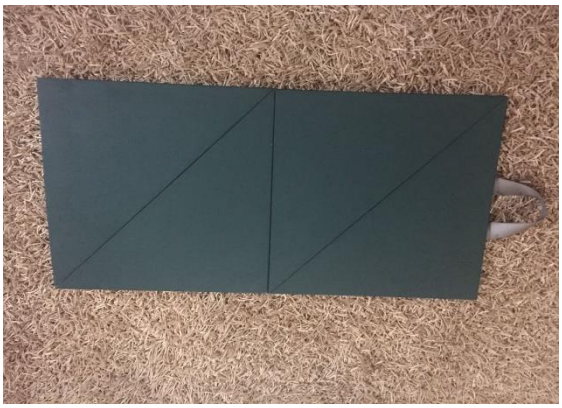
Disassemble the diagonals from the rectangle and create a parallelogram.

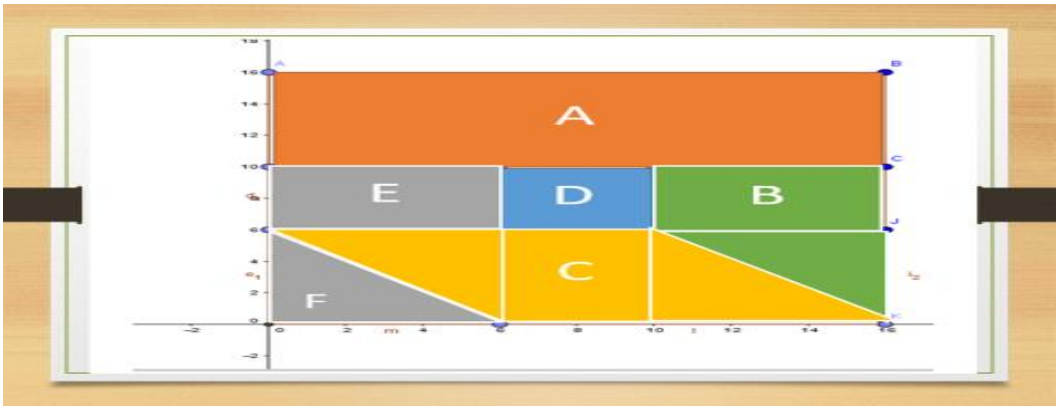
1. What can be said about the angles of the parallelogram? Use a model of the parallelogram from cardboard so as to investigate the relationships between the angles?
2. Are the diagonals you used in the rectangle suited for the parallelogram?
3. Do the diagonals bisect one another?
4. Are the diagonals perpendicular to one another?

Project: The <u>Orchard</u>. The students planned a teaching aid that illustrates the parts of the orchard – triangles and quadrilaterals that the father bequeathed to his sons according to the story they wrote. A relative of one of the students who is a carpenter built the teaching aid with the precise measurements given to him. When the teaching aid is closed, it is possible to see a square or two overlapping triangles. When it is opened, it is possible to create different shapes.

The model was used as a means of illustration for the solution to a complex arithmetic story, when at the end an Online game was held, titled 'Kahoot', with questions pertaining to the calculations of area of quadrilaterals and triangles, which constituted parts of the field that the father bequeathed to his sons.







Project: The <u>Angles Using the Clock</u>. The students decided to write a story that addresses angles, which is close to the children's world. In parallel, they built a huge clock, which constituted an aid for the learning of the angles in two ways, from both sides, so that it illustrated easily and clearly the angles. The model of the lock was built by one student's husband.

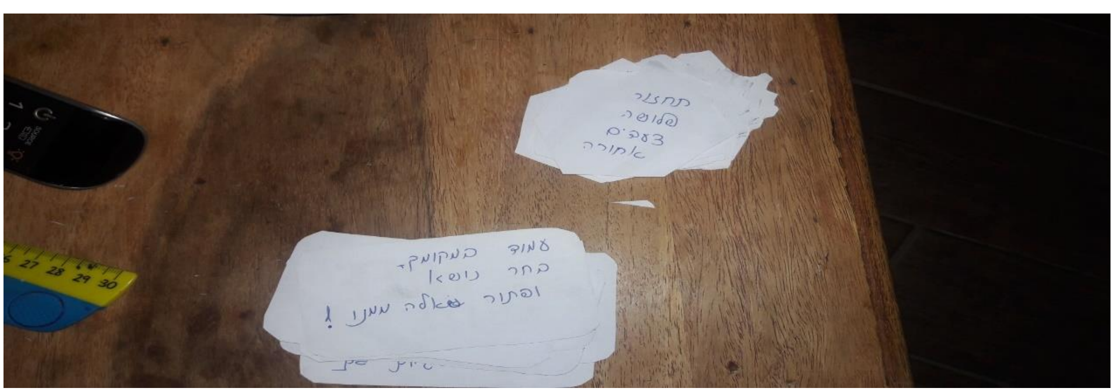
Other side of the clock in colors:

Project: <u>Multiples of 7 and 8</u>, A Story in Rhymes for the Practice of the Multiples of 7 and 8. The students wrote a story in rhymes that help the children remember the multiples of 7 and 8, which are the hardest multiples to remember.

Project: Monopoly of Three Dimensional Objects. The students built a monopoly game with clear instructions (cards of riddles that address three dimensional objects).

The sites are authentic and recall in their structure three dimensional objects – famous monuments from the world such as the pyramids in Egypt, the Tower of Pisa in Italy, the Blue Mosque in Istanbul, and so on.


FREE Gift
נקודת התחלה
הפסקה


עמוד במקומך
בחר נושא
ופתור שאלה ממנו!

Project: Fibonacci Sequence in an Olive Grove. The students wrote an imaginary story about an olive grove and the proliferation of trees according to the Fibonacci sequence. One of the students is a Druse student who lives in a village in the Galilee and has an olive grove. Every year, she is busy with the olive harvest and with the preparation of olive oil. She brought the authentic idea of connecting between the Fibonacci sequence and the olive groves.

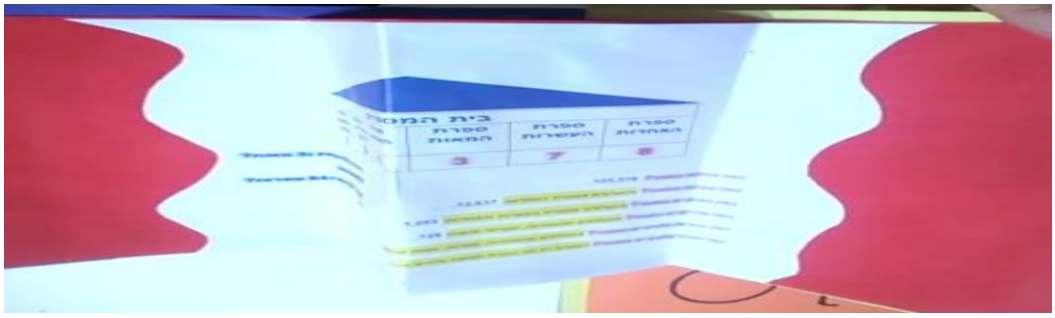
Project: The 'Exploding' Box (with Navigation Cards in Mathematics inside). Inspired by the romantic 'exploding' box with pictures of loving couples on the many sides, the original 'exploding' box was built, when opening on every side (and there are many sides) there is a different navigation code in mathematics for the fourth grade.

The product is relevant to the fourth grade pupils who are average or who are having difficulties who can be helped by the navigation cards found in the box. The box can be personal or group.

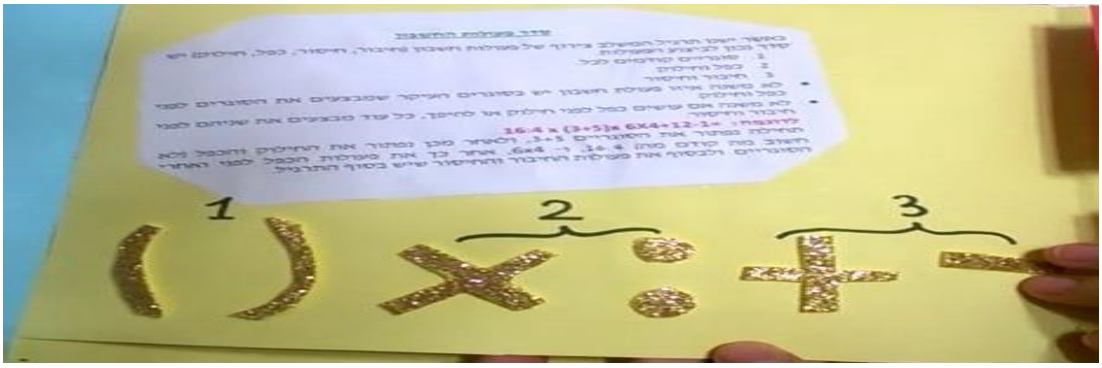
The navigation cards built addressed the decimal structure, the division signs, the properties of zero and one, powers, vertical multiplication, long division, and primary numbers, and factorable numbers.

Example of the navigation cards found in the box in the many sides.

House of Numbers

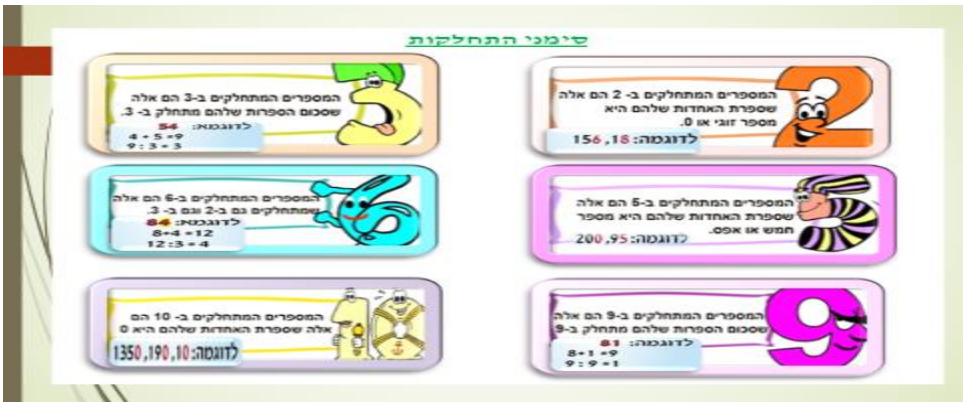


Order of Operations in Arithmetic



Factorable and Primary Numbers

Division Signs

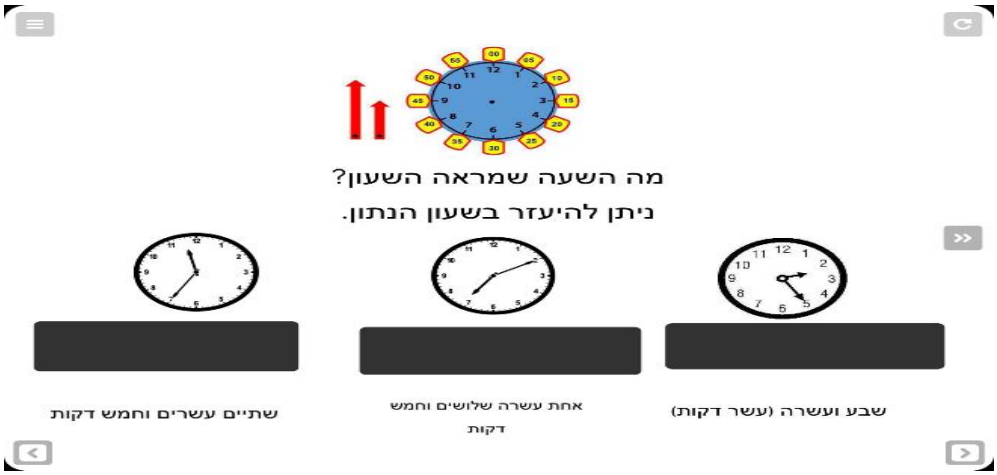


Project: Use of an <u>Application in Teaching the Clock</u>. The final product of this project is found in the Tiny Tap Application. To enjoy the product, it is necessary to download the application to the iPad. (It is possible also on a computer or another tablet but there the application works less well.) In the search line in the application it is necessary to write 'Tick Tock the Clock' and download it.

The children enjoy playing on the computer and on the iPad and through the story and the practice that the students built in the Tiny Tap Application the children will learn the skills of a clock in an authentic, experiential, and significant manner connected to their everyday life.

In parallel to the reading of the story and the accompanying activities, the children are supposed to write their own activity journal for at least one day and to describe their activities, the time at which they occur, and the duration of the activity. In addition, the students prepared a large clock for active illustration as an additional teaching aid.

Examples from the story they built using the application and the accompanying tasks that suited the story.



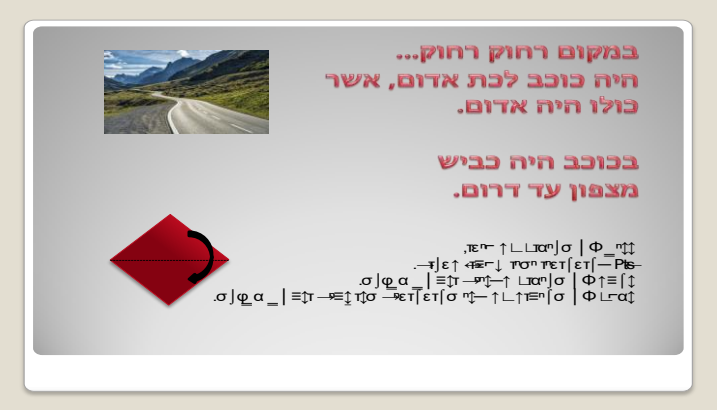
Suddenly Yossi asked his mother: Mother, how do you always know to arrange everything, when it is time to shower and when it is necessary to eat?

What is the time that the clock shows?

It is possible to use the given clock.

Seven ten (ten minutes) – Eleven and thirty five minutes – Twelve and five minutes

Project: The Red Planet – Origametry (Origami Geometry). The students wrote a story that combines geometric shapes through origami (paper folding). A number of slides from the story are attached. During the reading of the story and its description in a lovely presentation, the children folded paper they received according to orderly and graduated instructions and were asked about the properties of the shapes they obtained. At the end of the process, they obtained a shape of a red heart.



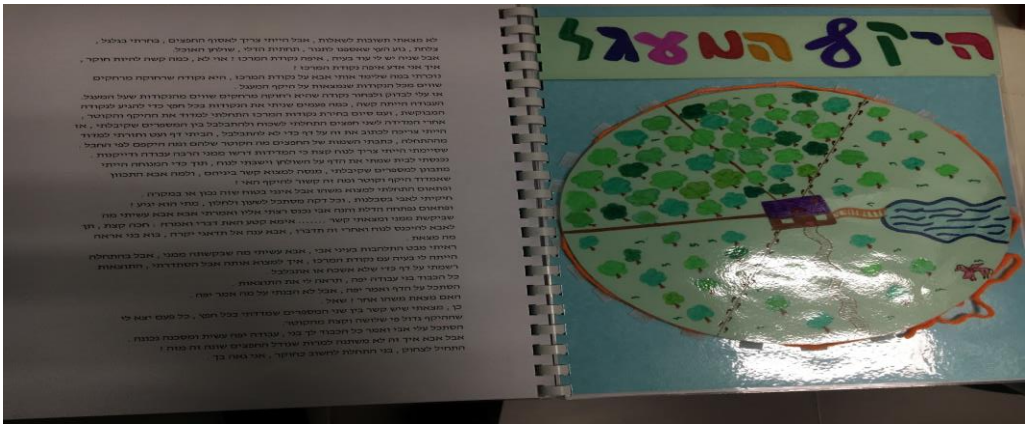
In a faraway place … there was a red planet, which was entirely red. On the planet there was a road, from north to south.

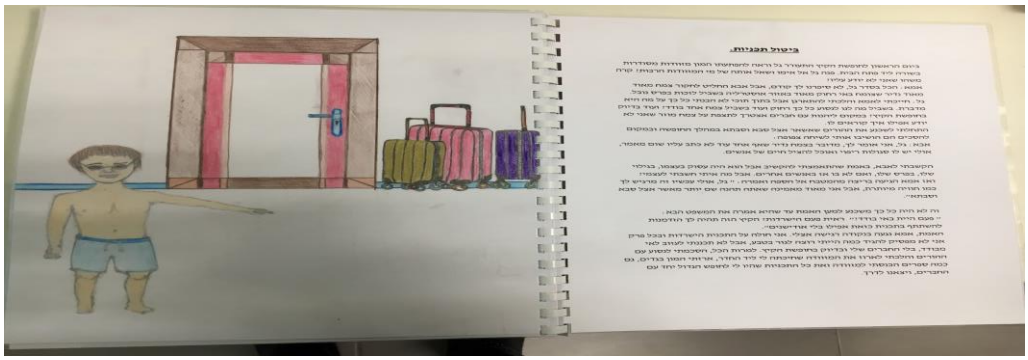
Place the square on its side, so that its vertices will go from north to south. Fold the square into two and open again. Perform the fold on the other two vertices and open again.

The product obtained at the end of the story:

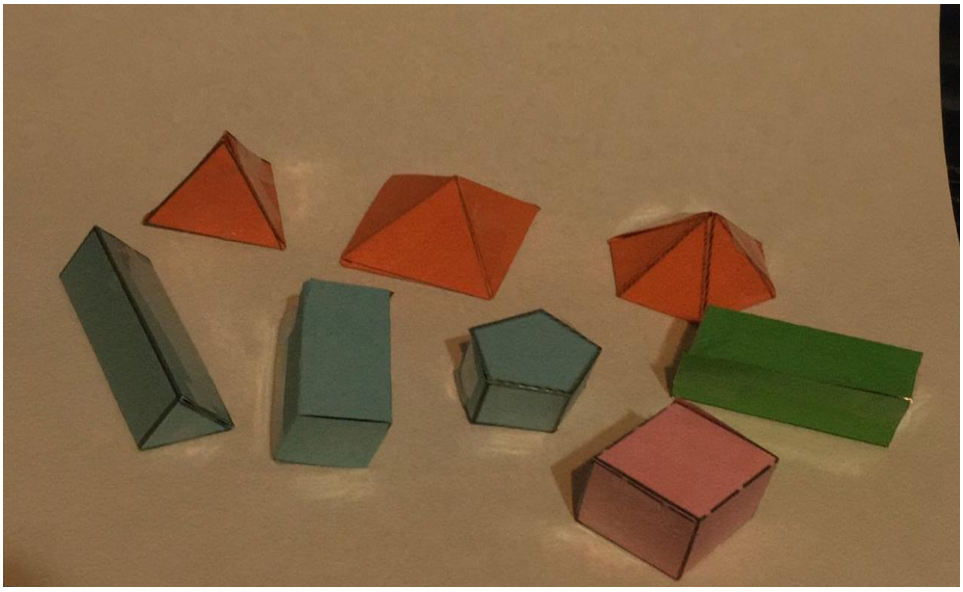
Project: <u>Gil and Circles – A Book on the Perimeter of the Circle and the Area of the Circle</u>. The students wrote a story about Gil, who moved with his parents to a distant and round island. On the island he undergoes strange experiences and during them he discovers the formulae for the perimeter and area of the circle (through the arrangement of the segments of the map of the island as a rectangle – see the last picture).

The child of one of the students illustrated the book.





Project: Revolving Three Dimensional Objects – Building a Game on Geometric Objects.



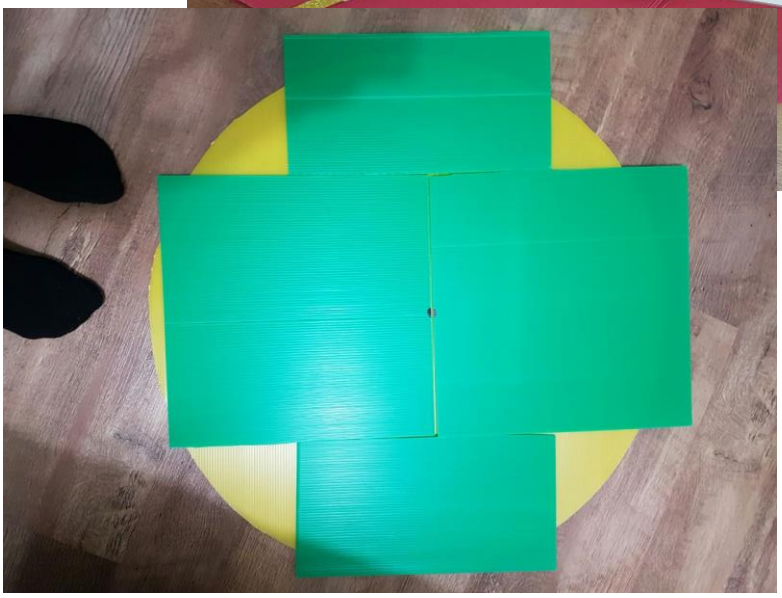
Project: The Kingdom of Polygons - The Princess Parallelogram. The students wrote a story in rhyme (in the original Hebrew) on the princess parallelogram and the knights of the rectangle and rhombus who attempt to conquer her heart.

A number of slides from the story and the activities that accompany the story are presented here.

When the king is seen, then it is immediately noticed that the King **square** is a shape with four vertices, 2 above and 2 below. King **square** has four sides and they are all equal, the sum of the angles is 360 degrees!

The king had a single special daughter and she is the Princess **Parallelogram**. Her qualities are many. Like her father, the sum of her angles is 360 degree. The Princess **Parallelogram** has a character of her own. In contrast to her father, she is not stable and sometimes tilts to the side. Her two angles are opposite and equal and her opposite sides are parallel, and most amazingly they are equal.

Project: Teaching Kit – The Circle (for the Teacher and the Pupil)



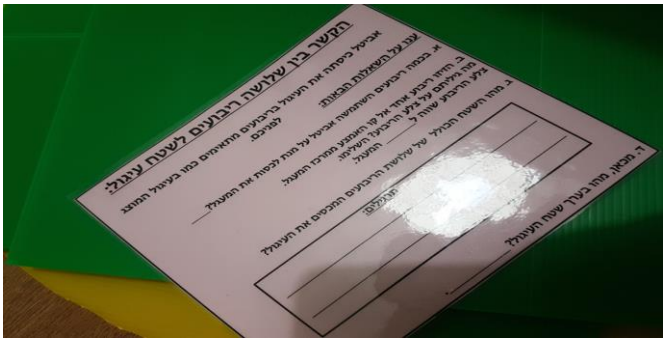


Table Number 1: Evaluation of the Mathematical Products

Groups	Required previous knowledge	Mathematical skills the product addresses	Mathematical accuracy	Authenticity/ creativity	Product effectiveness / usefulness	Product aesthetics	Product quality * Use of four levels of thinking[4]
Ratio Clips	Four operations with whole numbers, expanding & reducing fractions	Finding the ratio between quantities, finding the ratio between the part and the whole, finding the whole quantity or the partial quantities by a given ratio	5	5	5	5	5
Quadrilateral Family	Polygons, lines, angles, diagonals	Attributes of quadrilaterals	5	5	5	5	5
Orchard	Polygons, lines, angles, diagonals	Attributes of quadrilaterals	5	5	5	5	5
Angle in the Clock	Whole hours	Reading the clock, identification of angles, creating angles	5	4	4	4	4
Multiples of 7 and 8	Meaning of multiplication & multiplication table until 6	Multiples of 7 and 8	5	5	5	5	5
Monopoly of Three-Dimensional Objects	Traits of polygons	Traits of three dimensional objects: number of faces, vertices, sides	5	4	4	4	4
Fibonacci in the Olive	Sequences	Fibonacci	5	5	5	5	5

[4] The work program of the Supervision of Mathematics (2015) emphasizes that lessons in "mathematical thinking" should not be held especially. Rather, it is necessary to assimilate ways of learning suited to the cultivation of thinking in all the mathematics lessons at all levels. Level 1 is knowledge and identification of concepts and facts. Level 2 is performance of calculations based on routine algorithms (process thinking). Level 3 is the adjustment of the mathematical model of a routine verbal situation (implementation) and linkage between concepts, choice of strategies for solution (arithmetic insight). Level 4 is the open search for a way of solution, disassembly, and assembly (analysis and synthesis), research and explanation.

Grove		sequence					
The Exploding Box	The decimal structure and four operations in natural numbers	The division signs, the properties of zero and one, powers, vertical multiplication, long division, and primary numbers, and factorable numbers	5	5	5	5	5
The Clock Application	Whole hours	Reading the clock and calculating hours	5	5	5	5	5
The Red Planet	Quadrilaterals	Attributes of quadrilaterals	5	5	5	5	5
Gil and the Circles	Circle, area of the circle, radius, diameter, arc, sector	Perimeter and area of the circle	5	5	5	5	5
Revolving Three-Dimensional Objects	Traits of polygons	Traits of three dimensional objects: number of faces, vertices, sides	5	5	5	5	5
Kingdom of Polygons	Polygons, triangles, quadrilaterals	Attributes of quadrilaterals	5	5	5	5	5
Teaching Kit for the Circle	Circle, area of a circle, radius, diameter, arc, sector	Perimeter and area of the circle	5	5	5	5	5

Key:

1 – Low level - 5 High level

The results in table number 1 are according to my observations, the interviews, and the reflections and according to the reading of the written summative works. It appears that twelve groups of the fourteen groups received a score of 5 in all the parameters: mathematical accuracy, authenticity and creativity, effectiveness and usefulness, aesthetics, and quality of the product in terms of the four levels of thinking determined by the supervision of mathematics (Supervision of Mathematics, Ministry of Education, 2015).

In contrast, the other two groups, “Angles in the Clock” and “Monopoly of Three Dimensional Objects” received a score of 5 only in the level of accuracy of the mathematical product.

2. Two Main Themes from the Triangulation of the Analysis

Two main themes arose from the triangulation of the analysis of the written reflections, the drafts of the student – the "teacher", the observing students' observations and my observations, and the transcription of the interviews.

The first theme addresses the first research question.

1. What is the level of quality of the project and the products made by students in the learning process based on PBL?

The second theme addresses the three additional research questions.

1. What reflection do students have on the learning process based on PBL?
2. What kind of opinion do the students present about the socio-psychological environment they experienced during the learning based on PBL?
3. What kind of the student behaviors were observed during the learning process based on PBL?

1. 'Freedom in the framework' (Harpaz. 2015).

In the past three decades, in educational thinking and practice a new model of thinking and activity has steadily formed – the "third model" (Harpaz, 2015). The third model differentiates itself from the two models that preceded it, what is called "old education", in which the "curriculum is at the center" and what is called "new education", in which the "child is at the center". Harpaz calls the first model a framework, the second model freedom, and the third model "freedom in the framework". In such a model the pupil is active and involved in his learning in the existing framework and develops in himself the skills of the 21st century.

The active and involving learning environment through projects in mathematics as encouraging the development and cultivation of the skills of the 21st century: investigative and applied learning, relevant and authentic, critical thinking and formative

assessment, collaboration and belonging to the group, academic seriousness, and relationship with experts.

2. “Your approach, not your talent, will determine how far you reach.” (Zigler, 2012).

The empowerment of the student in active and involving learning through projects in mathematics – from fears, frustrations, and difficulties to curiosity, motivation, self-direction, and satisfaction.

3. The First Theme

The first theme is ‘freedom in the framework’ (Harpaz). The active and involving learning environment through projects in mathematics as encouraging the development and cultivation of the skills of the 21st century: investigative and applied learning, relevant and authentic, critical thinking, and formative assessment, collaboration and belonging to the group, academic seriousness, and relationship with experts.

In each one of the models mentioned in the first theme, I will conduct a discussion according to my observations of the quality of the projects and their products. In addition, I will present the students’ opinion, viewpoints, and feelings, with reliance on the quotes from the observations of the observer student in the group, the interviews, and the reflections they wrote.

Investigative and Applied Learning from the Author of the Research

Observations. All the groups worked on applied projects, since my instructions were to create some startup – an innovative and original product that would be a significant teaching aid for teachers of mathematics in elementary school. The ratio films were thus built, the different stories in a regular binding, in a presentation, or in an application, and the different teaching aids for teaching the clock, angles, geometric shapes, polygons, and the circle. The students applied and examined all the products that they built in their practice classes with their pupils in elementary school, were enthusiastic about their use, and were happy with their success.

In every group, it was necessary to investigate in-depth the topic and the specific mathematical skill chosen so as to search for effective and clear strategies for the teaching of the skill and to find an idea for a suitable, innovative, original, and meaningful teaching aid. The students investigated the mathematical skills and the teaching strategy suited for it through articles, textbooks on the topic, and ideas from additional Internet websites. They discussed, explained, and solved in the first lessons. The active inquiry continued in all stages of the planning and construction of the product, and included trial and error and experience in actuality in the school classes.

From the Reflections, Interviews, and Observations of the Students:

The students felt that the teaching aid they build best illustrated the traits of quadrilaterals and will contribute to the pupils' understanding.

"Our pupils can implement the kit of aids where they already learned about the properties of the quadrilaterals and can see the properties with their very eyes. For instance, they can see the sum of the adjacent angles using the cutout model of the parallelogram."

The students felt they were active and were satisfied with the product they built and learned how to guide their pupils in the building of their own products.

"I think that the course greatly benefited me. This is an enjoyable and activating course; it is not a dry course of practice and test. In the course we learn the pupils' viewpoint. During the course we studied, we worked very hard, but we were active in every lesson. I am very satisfied with the process and with our product that can serve us as teachers. I teach my pupils according to the project- based method and I will let them build a product by themselves that includes, for example, films in different areas and not only in mathematics."

The application that was built contributed to the students' motivation to continue to research at home.

"The application inspired not only great motivation, but also learning that is not in the framework of the studies. Part of our goal in the application is to get the child interested in everything connected to the topic of the clock and for the child to continue to investigate at home."

The students were enthusiastic about the use of the application and the opportunity they had to display creativity and to upgrade an existing product.

"Since we, as new teachers, try to constantly challenge our pupils, it is very important for me always to integrate technology in my lessons, and therefore, I am already waiting to enter our third grade classes to teach the clock through our application after already having shared it with the homeroom teachers. What I liked in the application is that it is always possible to add additional material, using not only the application itself but also adding my own personal input."

"To deepen our knowledge on the topic of quadrilaterals and to get ideas for concretization and realization, we based on two articles, on textbooks, and on different Internet websites."

In addition to the search for sources of information and the reading of these sources, there was a thorough examination of the materials from which the product would be created, until the most effective materials were found.

"The kit of aids for the teaching of polygons that we prepared was not designed ahead of time for the use of the Polygal (polycarbonate) material and divided pins in the materials. In essence, we tried, we examined, and we researched a number of materials such as pipe cleaners and a material called Mapal and we were not satisfied with their illustrative possibilities, until in the end we reached the final product with the Polygal material."

In the project of the development of an application on the topic of the clock, it was necessary to ascribe greater weight to technological learning and less to the engagement in mathematical skill.

"Our active inquiry less focuses on the topic of the learning since there isn't much didactic material to learn about the clock but the inquiry is on the technological knowledge. I was exposed to the application, to its advantages, to the different possibilities. For me it is very important since I see in the class a number of children enjoying the computerized tasks that I give them such as obtuseness or tasks on the" Horizon". Then if I have the possibility of preparing such a task that will exactly suit the topic that I teach, this is wonderful."

The products were developed and improved following further study of the topic, writing many drafts, feedback and activation of the product in some of the classes in the school.

"As the time passes and our inquiry in the topic deepens, we took the product we aspired to do and improved it – we repaired it using many drafts, and through the experience of the four members of the group who taught the lesson in their practicum classes. Their experience contributed greatly to our project – to understand where we need to change, to add explanations, to delay, how to end the lesson, whether to let the pupils copy the concepts and the definitions from the blackboard ... or to be satisfied with the handing out of a page of concepts at the end of the lesson."

"I felt that I must attempt to convey the experience to the class where I am having the practicum – the third grade. I was curious to see the students' response and for this purpose I made a lesson on the topic of polygons."

The in-depth reading and study of the topic contributed to the feeling of confidence and willingness to teach the mathematical topic with great professionalism.

"The personal inquiry and the intensive work on the topic caused me to understand the topic in-depth. I now discovered there are more than a few methods of learning for this topic and this is a topic a bit complicated for the pupils. Because of the contents I read to enrich the knowledge on the topic we chose, I felt professional and ready for the field."

Relevance and Authenticity Were Seen in the Work on the Project and in the Products

From my observations, the guidance given to the students for the work on the projects was open and vague, but obligating authenticity and relevance, since they were asked to become 'startup founders' and to prepare an innovative and original product that would be an important teaching aid for teachers of mathematics in the elementary school in any mathematical skill that they would choose. All the products built were relevant and meaningful for the mathematics teachers in the elementary school and for their pupils. In essence, the mathematical skill and the idea for the product received my approval only after I examined their relevance for the pupils.

The products were relevant to the teachers of the elementary school and their pupils and illustrated mathematical skill from the curriculum for the first to the sixth grades. However, not all of them were authentic in their nature. The kit of teaching aids for the calculation of the area of the circle and the perimeter of the circle is the most prominent example of a project that is not original. The students were helped by means of illustration existing and known to the professional mathematics teachers. The sole originality that was seen was their increase to the level of illustration in the frontal teaching.

I expected of them at least an addition of some uniqueness in addition to the known illustrations, but these too were foreign for them and they needed my inculcation. They were surprised and enthusiastic themselves. It was possible to see that in their group there were gaps in the knowledge and lack of control of the chosen skill and in addition the student who was in the role of the teacher was not a sufficiently dominant figure to create an original and meaningful product.

The Revolving Three Dimensional Objects and Monopoly of Three Dimensional Objects Groups asked different riddles in play, but I could not know whether the riddles are original or were taken from the Internet. In any event, I expected that the riddle would be on the level of higher thinking, and in essence they were on the level of knowledge and understanding and not beyond this. These groups too lacked mastery on the topic of

shapes, and therefore it was not possible to discern dominant figures that would lead the group members to in-depth places.

All the rest of the products were authentic, such as the illustration of the orchard from the tree, the clock application, the exploding box that includes navigation cars for the fourth grade, teaching aids for the family of quadrilaterals, collection of the ratio films, the red planet in origametry, and the original stories written on the family of quadrilaterals, on the clock, on the multiples of 7 and 8, and on the circle.

From the reflections, observations, and interviews of students:

The fascinating story written on the polygons had great color and rhyme. The students believed that because of it the traits of polygons will be learned well and will be etched into the children's memories.

"The topic was relevant to the elementary school beginning from the third grade, since the pupils begin to learn about polygons and their basic properties, and every year a little bit more knowledge is added to the knowledge they acquired the year before. I believe that when the pupils will see the presentation for the first time after the continuous learning they will learn the properties of each one of the shapes because of the colorfulness and fascinating rhymes found in it. Thus, beyond the relevance to the pupil, the story will make an impression among the pupils, a type of learning experience that will be engraved in their memory for years."

The members of the group that engaged in the circle were enthusiastic about the product they received and assessed it as excellent. However, they were impressed by the idea for providing the calculations of the area of the circle and the circumference of the circle, through exploration and discovery that they did not experience as pupils.

"In my opinion, in the end we reached an excellent final product, which exemplifies and explains to the students in the best possible way that there is the way to calculate the area of the circle and the perimeter of the circle. I am happy that we prepared such a model, it is different from what I as a pupil and as a teacher in the future knew."

The Orchard Group built an original product from wood that illustrates a large part of the quadrilateral family. They deliberated greatly in the planning and development of the product and were helped by a carpentry expert (carpenter). In the end, a product with great meaning for the teachers and their pupils was obtained. The students decided that they wanted to register for a patent for the product.

The draft of the teacher in the Orchard Group: "*First I asked the group members to concentrate on what shapes we will focus on from the quadrilateral family. We decided the rectangle, the square, the trapezoid, and the parallelogram. Here the idea was born for our amazing product. We wanted to build a model of shapes from a rigid material with which it is possible to play. At this stage we did not know how it would look and what really is it possible to do and therefore we began to seek the advice of professionals. Rova had the amazing idea to create a sort of cube (square) that when we open it would become a rectangle (that is divided into two triangles) and when we open only one triangle from any side it would become a parallelogram and when we close one triangle it would become a trapezoid. At first we thought about the cube, to be of cardboard, and then we thought of glass ... we wanted our projected to be tied to the real world, to the world of the child in the school. We thought about something that we can use and be helped by it as teachers. If already then already ...we searched also to find content in which there is some sort of meaning in the eyes of the students and we reached the understanding that this must be something visual and authentic and having a certain value ... we turned to Dua's uncle, the carpenter ... Dua updated us at this stage that the product is progressing with her uncle ... and she further updated us that the material from which the product is made is wood, like a board that can be written on with chalk and erased ... and we were very satisfied."*

Critical Thinking and Formative Assessment Were Seen throughout the Process

From my observations, the multiple drafts written by the students in the roles of the teacher, the written reflections, and my feedback in the sessions with the group members and the feedback of the students from the other groups in the assembly constitute a main instrument in the formative assessment and a time framework best for

critical thinking. Following the summative event of every group, it was possible to again awaken critical thinking and to evaluate summative assessment the project and the product.

In addition, experts in different fields of knowledge, such as the field of computerization or the Hebrew language, gave formative assessment during the process. I was witness to this feedback; however, the results were good after the encounters of the students with them. They mastered the application and were precise in the Hebrew language.

In essence, the evaluators and providers of feedback were everybody: the students in the feedback in the assembly, the student in the role of the teacher, the student in the role of the observer, the lecturer, and the additional experts.

The students evaluated their work in every stage through conversations, asking questions, and feedback for one another. However, they examined the products in their experience classes.

Gaps in knowledge and less display of critical thinking caused them in the process of the building of the product to repeat their way and to correct, sometimes with mental aggravation. Thus the students from the Ratio Group corrected themselves when they discovered a mathematical error or an error in the name of the number in the film they filmed. They filmed again, although this process took from them sometimes the afternoon recess in the college. In addition, prominent frustration was felt among the students of the project of the kingdom of polygons when in one of the feedbacks together they were given criticism on the rhymes in the story they wrote at the expense of mathematical accuracy.

The students in the role of the teacher constituted an important factor in the formative assessment. Also, the observer who was a student saw from the side the discussions and could evaluate the situation temporarily in an objective manner. There were groups such as the Ratio Group, for instance, in which they decided to change the observer students in a rotation from week to week, so that everybody would experience

the looking from the side, the critical thinking, and the evaluation that is not made possible in active participation in discussions in the group.

The summative event again invited critical thinking of the observers and summative assessment. The great excitement during the presentations because of their tremendous pride in the product they prepared was apparent. In addition to the presentation of the product, they were required to tell about the process that they experienced in terms of different parameters. I invited to the summative events the president of the college, the head of the department of mathematics, the active and involving learning coordinator, and my colleagues from the department of mathematics. They were impressed by the presentation, commented, asked questions, and expressed their opinion. The peer learning was even felt at its peak when at the request of the active and involving learning coordinator my students prepared the process that they experienced and their products in one of the lessons to her students from the department of special education.

From the interviews, reflections, and observations of the students:

From feedback to feedback, the students began to think in a more focused manner and through guiding conversations and critical thinking they developed they could be more accurate in the mathematical skill.

From the reflection of one of the groups from the kingdom of polygons: "*At the start of the work it was hard for me personally to think critically and mathematically but after the first session with the lecturer who opened my eyes to all sorts of nuances, my manner of thinking and the thinking of my teammates changed and became focused, we began to think about every sentence we wrote more critically and on a high level of mathematical precision."*

The members of the group that engaged in the development of the clock application used critical thinking when they initiated the innovated idea that entailed technological difficulties both in the construction and in the application. In the end, they

decided not to give in to these difficulties and they created it and found suitable solutions effective for coping with the difficulties.

In one of the drafts the student who was the "teacher" of the Clock Application Group indicated critical thinking: *"There was some lack of agreement on the topic. To prepare the application it is necessary to work on the iPad itself, and therefore not everyone can prepare our product and in addition during the time of the teaching in the classroom we did not know how to overcome the problem that not every child has an iPad. We deliberated whether we should prepare a product that will not be used because of a technical issue. We decided that such an original and innovative idea should not be surrendered and that like applications are used in the classroom with a central projector, in teaching with our application we will connect a central iPad to the projector and a pupil who wants to participate will come to the front of the classroom and will operate the teacher's iPad.*

The students were helped by what they learned in previous mathematics courses and by lecturers from the college who could advise them in terms of proper Hebrew language.

"The digital book was written through the story that we wrote in the style we learned in the course of story and mathematical rhyme with Miri. Our relationship with an expert in this stage was when we sought the counsel of Dr. Raviya Basis, a lecturer in the college on the topic of language. I sent her the story for her to review and perform a correction in terms of language and in social terms."

There were many junctures of evaluation, whether these were internal feedback in the group or external in the class with my guidance. These feedback sessions contributed to formative assessment, to the possibility of thinking and analyzing the drafts, changing and improving. All these contributed to motivation, and they were what shaped in essence the continuation of the process and the final product.

"Throughout the entire process, we had many junctures of assessment, whether this is in the presentation of the product to our lecturer or to an outside factor so as to

obtain feedback and every time we did this we received much motivation to continue the process, especially when we saw that we are succeeding in advancing and the direction is positive."

The implementation of the product in the practicum classes during its preparation also constituted formative assessment.

"We had one meeting with the group and Miri in the middle of the project. From this meeting we made a final decision about the initial product. In addition, we tried the kit of aids in our practicum lessons, and we saw what is more practical and what is not."

Sometimes in feedback in the plenum certain groups receive criticism that was not simple, which created great frustration. My direction and mediation, the cohesion of the group, and their commitment to the process helped them overcome the frustration and return to work with full energy.

From the reflection of the Kingdom of Polygons Group: *"A difficulty we encountered during the semester was in the first showing of the presentation that we built before the entire class. We came to the presentation ready, confident in our selves, filled with enthusiasm from what we did and our product. Shortly after the beginning of the presentation, we received very unpleasant criticism from the classmates (while the criticism was correct, it was very difficult to hear it). This dropped on the group members the difficulty with continuing and frustration. What lifted us up to continue to work and to correct according to the comments with full strength was the instructor's mediation, the group cohesion, and the commitment of the girls in the group to one another. In the end, we built a very invested project, intelligent, creative. The final product – the presentation, is very colorful, rhymes, and has entertaining characters that the pupils will be curious to know."*

The criticism gave us an unpleasant feeling but in retrospect it is what caused our work to be more accurate mathematically.

"Throughout the work, there was one time that we were asked to present before everybody, in front of the class, our situation at work. We came with great confidence for

the presentation, but the criticism we received in the assembly was not so pleasant and was even difficult to hear for us. This slightly decreased the group confidence, but among the members of the group there was great support, and it was said that we won't let this influence us and we will attempt to understand what they intend and to draw conclusions. The criticism we received in front of everyone was something that in retrospect helped us make our work better than it was in the beginning, in terms of the mathematical language becoming professional and correct."

The students in the role of the teacher constituted an important factor in the formative assessment.

"The student-teacher is filled with motivation, arranged and organized in terms of the contents and explanations, takes command, and conducts a division of roles for the group so as to promote the project."

The student in the role of the teacher in most groups evinced great responsibility and abilities of leadership and navigation in the internal feedback in the group.

"The student who was in the role of the teacher took the role with the full responsibility and seriousness and directed us frequently when there were different arguments, and thus we decided together what we are doing onwards in each and every session. We knew that we are supposed to meet the time schedule and that each one of the girls in the group needs to take responsibility and be mustered and to help so that our project will succeed, and this is how it was in each and every session until we collected a range of ideas for the continuation of the project."

The role of the observer also was constant but the students thought that it is preferable to perform rotation in this role so that everybody will attempt the experience of the observation from the side. They maintained that the behavior of somebody who experienced the role can change for the better.

"One of the insights that we reached during the project is that the role of the observer should be performed by each one of the members of the group, so that we all

can experience this place and understand how the group is managed from the side and in addition that each one can take part in this role."

The observers sometimes could not intervene and participated although they were in group discussions. They praised the role of the student who served as a teacher and saw his intention from session to session to collaboration and achievement of the objectives. They told that as observers they can see how things work out in a group, what needs to be done and what not.

"In this project I served as an observer, but of course I was also a part of the members of the group and I participated in discussions. My role was to document the course of the meetings of the group members and to send to all of them at the end of the meeting what we needed to do and to prepare for the next meeting. As the observer, I succeeded in seeing how a teacher helps and directs the students and how the students cooperate and together succeed in achieving the goals. When standing on the side different things are seen that cause you to understand how things are managed and what needs or does not need to be done."

In this reflection the student describes the summative event as most successful. He maintains that the very transmission of the origametry product with mathematical accuracy in an experience of interest and motivation for all the participants shows their academic depth in the mathematical skill, their transformation into content experts, and the collaboration in the group.

"First, I will say that I greatly enjoyed conveying our product to the class. In my opinion, our teamwork and cooperation helped us greatly in obtaining the present product. My great excitement derives primarily in light of the fact that we as a group invested considerably in the learning of the academic material that our project addresses. After the in-depth research of the topic, continuous learning, and our many encounters and discussions as a group together, we presented the final product in an appropriate and professional manner. Our presentation in front of the class was the implementation of our learning – it was possible to see that my group members are professional in the topic and know what they are talking about. The very fact that we

succeeded as a group in conveying the course of the project and in implementing the product of the origametry without mistakes, in a manner that is clear to the class, through the curiosity of the students and lecturers in the class and their enthusiasm, reflected to me personally that the inquiry work that we did was sufficiently in-depth and successful."

Collaboration and Belonging to the Group Were Seen throughout the Process

From my observations, my guidance for the work on the project was open and rather vague intentionally, so that some of the students were a little afraid and felt insecure. However, I enabled them to build teams freely. These facts caused the students to collect into groups with their familiar friends who in any event sat next to them, and thus their self-confidence rose slightly.

All the students were organized, met the time schedules, were responsible, and respected one another. Even if there were differences of opinion, they were resolved easily.

The planning of the work in the different teams was influenced by the charisma of the student who was in the role of the teacher and from the knowledge of the students in the mathematical skill that was chosen. As the teacher was more charismatic, the division of roles in the group was more effective and the progress was the best. Even if they were not experts in the mathematical skill, when the "teacher" had leadership abilities it appeared that he manages their learning on the topic of the division of the articles to each one of the members of the group and responsibility was given to each one for the article, and the continuation of the work also flowed in harmony.

As I noted in the previous chapter, collaboration was felt also with my colleagues and the students from the department of special education, when I invited the college president, the head of the department of mathematics, the active and involving learning coordinator, and my colleagues from the department of mathematics to the summative events. They were impressed by the presentation, commented, asked questions, and expressed their opinion. The peer learning was even felt at its peak when at the request of

the active and involving learning coordinator my students presented the process that they experienced and their products in one of the lessons to her students from the department of special education.

From the interviews, reflections, and observations of the students:

The first quote shows how the students assembled into groups according to seating positions and friendships.

"In the first lesson of this course, I understood from the lecturer that in this course we are asked to prepare a work with a number of girls in the group. The groups in the class were divided really according to our seating place the day."

The concerns about cooperation are described in the words of a student from the Ratio Group, which consisted of eight students:

"*After we got along in the group and decided that we would work together, we began to think about a topic from the field of mathematics that is suited to everyday life. After we did this, fears arose in me. How can we get along in the group with so many girls? Will the girls respect one another? Will they not laugh if somebody says something that is not correct or related? Can I say freely what I am thinking? Will each one really contribute of her time and do what is assigned to her (there is in every group eight girls)?"*

This student saw that all her fears were for nothing and that the cooperation was its best. Other students said similarly.

They noted the respectful way in which they behaved towards one another, the coming to meetings on time, and the bringing of the required equipment to every meeting, without forgetting.

"During our meetings, we realized that all the students come on time, bringing what they were required to bring and not forgetting, behaving in an adult manner, and respecting one another, seeking advice of one another, speaking in an appropriate and

respecting manner, and really it is possible to say that we drew closer and learned to know one another better."

One of the students was surprised about her friends' responsibility, since they came to the meetings with great precision and perseverance, as if they come to a lecture.

"*The work in the group was enjoyable and fun, and all the students contributed to the work, did what they were assigned to do. I was surprised that truly they all care and all come to the group meetings as if this is a lesson with a lecturer, and this is compulsory, but since everybody truly cared and it was important to everyone we came and we worked in exceptional cooperation."*

Everyone can express herself appropriately and contribute her part to the group. Collaboration, respect, attention, and consideration of one another were apparent throughout.

"We as a group worked with wonderful cooperation. When each one knew her part and contributed it to the group. Each one simply accepted the other and thus in essence sentences were composed from the story, and the story as a whole."

"Among the members of the group there was communication in a respectable manner. Each one could express herself, and the rest of the group members listened without disruption. It was possible to see the full cooperation of all the girls of the group from the beginning of the project and until its end."

The cooperation contributed to the group members' motivation, and even if there were some differences of opinion, they were easily resolved.

"In our group there was full cooperation among all the students who motivated one another. There were slight differences of opinion and different deliberations, but we solved them quickly."

To every meeting they came with a formed goal, ideas, and objectives for the continuation, and this focused them in professional terms and contributed to the meeting

of time schedules. In the first meetings every one of the group members came with ideas from an article he read.

"*We came to each meeting with a clear goal, which helped and directed us both in terms of the time and in professional terms."*

"After the first session, the fears and the pressure begin to decline rapidly, we divided the roles so that each one can express her opinion and contribute her part. In the first sessions, we read the articles that each one brought and slowly we began to crystallize the sub-topics and to work on the project."

Frequently corrections were necessary, but the collaboration worked well, without frictions.

"We began to film the films according to the topics, for the most of the topics there are two films built from the inculcation of the topic and the response to the question. We filmed, we stopped as necessary, and sometimes we filmed the film again because of different mistakes. There was cooperation between all the students. There were no arguments, we agreed with one another."

The division of the roles was undertaken according to the abilities and capacities of each one of the members of the group.

"*I personally found it to be a good and challenging experience. This way I better knew the members in the group, in every problem there was we shared with one another, each one saw her abilities be expressed (art, presentation). The work process on the project taught me much about the cooperative work process, the required teamwork, the proper communication among the group members, the willingness to contribute and to take responsibility for the parts of the project as well as the project as a whole. I learned that each person in the group has something to contribute and the greatness of the shared work is to discover what each one of the group members can contribute and to enable this ability to be expressed during the project."*

The division of roles in the staff was clear already from the beginning, and each one could have expressed himself and contributed according to his abilities.

"The division was very clear, and each one went to her natural place. A person who is good in writing – thought about the story (a quick reminder – we illustrated the series while using a story in the presentation). The person who is good in design – me, I built the presentation through the shaping of the characters and objects suitable for the project, and a person who is good at summarizing material – summarized the articles."

The following quote describes the division of the roles as seen in the group that addressed the development of the application of the clock.

"Each one of the participants used her abilities and the technical means at her disposal, such as the iPad of Yamit (in the continuation in the role of the 'commander' or the 'teacher' in the team) and another device of Samach (in which the application 'Tiny Tap' was installed, in which we built the backbone of the product/project), the laptop on which Inbal (in the continuation the 'documenter' in the team) documented every conversation and idea that was formed, and me, Tzachi, I used my graphic abilities and my personal computer, Tirza, Meital, Ranya, and Inbar, each one used a smartphone and aids brought from different sources on the clock, background materials, presentations, and development of means that together we decided how to incorporate them into the final product."

The observer of the group that worked on the ratio reported harmony between them. Nobody was afraid to ask questions or to express ideas, even when there was chattering that was unrelated they immediately returned to the ongoing work.

"The group conducts a spectacular dialogue so as to form a decision pertaining to the activity they chose to do, one raises an idea and the other has an idea that complements it, thus creating harmony an agreement in the group. There are a number or girls who are focused on the small talk and do not appear so concentrated. Periodically there is a return to attention to the interesting project on which we are working."

Periodically humor was felt in the group, and this contributed to the increase of the motivation among the friends to continue the work.

"In the group there is no shame at asking questions and expressing the opinion. Karin asks questions and seeks to know what examples will be given. Gal responds pleasantly. There are moments that the group incorporates humor that gives them motivation to continue and to act."

For the improvement of the cooperative work and from the understanding that the observer in her role sees things from the side in a clearer manner and thus makes more conscious the group processes, it was decided upon rotation in the role of the observer in the Ratio Group.

"We very quickly understood that the role of the observer is an important role and it was decided to divide it between the members of the group when each meeting it will be somebody else."

The Orchard Group decided upon clear rules for cooperative work, after three of the students came from another group in which they felt especially unpleasant thoughts. It is necessary to also add that the members of this group were from the academic retraining, Jews and Druse who did not know one another beforehand.

"*In the first stage of the work, in the groups we defined clear goals from the beginning such as every student takes part in the group activity, and we said that the work in the groups is a way of work and a way of life, cooperation, tolerance and patience are our milestones, we listen to one another from the acceptance of the difference, and even if one or another idea is not accepted by the members of the group we will investigate it and we will reject the idea only when the person who proposed it understands herself that the idea is not the best. The division of work was organized and each one understood what was assigned to her, nobody was contemptuous of the tasks and the entire project."*

Three of the Orchard group members moved from a group in which they were not treated with respect in their opinion. Therefore, they initiated an agreement that obligated

the members of the new group – the Orchard group, which includes basic rules of behavior.

"It should be noted that three of the members of the group were in another group before, and since they did not have a good name and everything described above happened to them they decided to leave the group and to create a new group. We chose to define clear rules for work on the project and they are: we listen to one another, we share ideas and information, we talk quietly and respectfully, we let each one of the members of the group express her opinion, we let each one in the group take an active part in performing the task, we give into one another, we compromise as necessary, we are helped in abilities of each one so as to perform the task effectively, and last we strive together to achieve the goal. In the entire process, we understood that we learn to take responsibility for our actions and we learn to maintain a schedule. And most important, in our opinion, we learn that a group always achieves more than does the individual."

The student who was in the role of the teacher in the Orchard Group understood that heterogeneity is not a problem and describes the great secret of cooperation in the providing of the comfortable place and a say for each one of the group members.

"I did not know anybody beforehand in this group and I asked myself how am I going to prepare a project of such a scale with people I do not know? What direction will I choose? Who will be my group members? And so on … Slowly the picture became clear, the moment we had a group that is formed and it should be noted that the composition of the group members is especially heterogeneous, we are different from one another, but this difference and the definitions we defined for ourselves of work in a group prepared us good for what is coming. I will say that I personally had a good and challenging experience, the dynamics were excellent and filled with positive experience. In every problem or dilemma that arose in the group, we shared with everyone and each one was a regular member in it, everyone is expressed and the reciprocal respect was preserved between the members of the group, including the tone of speech. I think as a teacher of this project and from a different perspective I saw that the different combination and the difference between us worked well and to our benefit although I

admit that in the beginning I was afraid of it, that the differences in the cultures will be expressed. I was happy that they were expressed but in the best possible way, I think that in addition when there is a dialogue of attention and shared learning and brainstorming occur in the sessions, there will be good products. In the group the moment that each one is expressed and each has her place and her say, this causes that I have very great internal motivation, great drive for the success of the project. I felt that I have somebody to work with, and the reciprocal cooperation occurred immediately.

The students were surprised at the end of the process, when they understood that they did not experience crises and that the multiplicity of the cultures and the lack of prior acquaintanceship between them were empowering.

"I was surprised to find out that we did not experience crises and difficulties during the project, something that was not obvious in the group work. I assumed that the reason entailed our desire to succeed and to bring a good product, suitable for presentation. We displayed considerable maturity in that we did not engage in discipline and unnecessary disruptions. I learned about myself that I should not give up group tasks, even if these are complete strangers, and it is possible to say even that there is a significant advantage in that I did not know the group members beforehand."

Cooperation and the best teamwork, pleasant and effective, are not necessarily dependent on the relations of friendship and homogeneity, as described by students from the Red Planet Group and the Fibonacci Group: *"I personally was not a friend of all the girls of the team. Of course we knew each other but we were not in a relationship of friendship. The project brought us close and united us. Moreover, what characterized us as a group is cooperation, each one received her place. Furthermore, we listened to one another, we gave advice, we developed all sorts of ideas, and we truly did not eliminate a single one. This is not characteristic of a group of girls of such scope (7 girls) but the support and understanding between us was so good that we overcame the difficult of a number of the girls. Although the dynamics was very good, there were still a number od difficulties along the way. The main difficulty is that it always was the not everybody was presented in all these meetings. In addition, of course there were arguments and crises of*

one kind or another, but we overcame everything as a group and this is what brought us closer and united us even more."

In the Fibonacci group, they reported that some of the group members were less active, but the ethnic difference is actually what shaped and improved the process and the product.

"On the level of the group, the topic was interesting and all the members of the group were curious about it, and it was apparent that everybody wants to contribute and to help. Every session began with brainstorming, when during it we discussed the chosen topic. How should it be presented? What products do we want to obtain at the end of the project? Naturally such ideas and others were raised and after the discussion left on the 'editing room floor'. For instance, I suggested that the story be in the literary genre, like a short fantasy story for children, and the other group members preferred to maintain a scientific theme. In retrospect, I agree that the chosen form of presentation is a winning choice. Some of the group members were more active verbally and some were less, according to their nature, but each one could express herself and obtain feedback for her words. It is possible to say with certainty that we would not have succeeded in reaching the present product in another composition and the ethnic diversity led to the choice of the final idea for the presentation of the series, an olive tree (Fibonacci). In addition, the process became easier during the way and the sessions with the lecturer contributed greatly to the understanding and the positive feedback."

The students describe on the one hand an easing in the work when they bear the burden of additional members and the responsibility divides among all of them and on the other hand they understand that they are assigned the collective responsibility and they cannot underestimate since their work influences others (the score of their course).

"*In the end, I discovered that this is a project filled with advantages since you learn much about yourself, you learn how to work with a large group, how to behave with them respectfully and to accept the opinions of others, but also not to give up your opinions, you share the work among many people and at the end of the process a great product comes out that you could not succeed in doing alone in such a short period of*

time. Moreover, when you work in a large group you must really invest and do this the best possible way since you have greater responsibility, this is not only your grade, this influences everybody and therefore you must be more responsible."

Some indicate about themselves that the work in the group suited them more than independent work and that they understand that such a product can be created only through teamwork.

"It can be said that if I did not have shared work, I would hypothesize that it would be terribly difficult for me to finish the project alone, and I would have very many difficulties with it."

The implications of their work with children: the students believe in this way and declare that they will implement it in in the field.

"*Already today in my experience as a student from the first year I let the pupils work in pairs and in groups since I think this is very important to help one another, to know one another better. Nevertheless, in the elementary school they are together six years continuously and this is something that is mandatory for every pupil to know the pupils in his class and not only their first names and their last names. Especially with pupils who are less close to one another, thus truly they have an opportunity to know in greater depth. Moreover, it is very fun to work in groups and to help one another since not everything falls on somebody but is divided equally between people, and this helps the pupils."*

The place of the student in the group: Like in every group dynamics, there is somebody whose voice is heard more and who leads the group and there is somebody whose voice is heard less. In general, there is no sense of passiveness among the participants.

"To the best of my understanding, in the work in the group the process of the work with the girls, not everybody had the same place and the same verbal and technological abilities and it was not always easy to express them but this was logical and reasonable for this to happen since in the large group dynamics that were not always

possible to anticipate are created, sometimes somebody took the initiative and leadership, for instance, Yamit, as a very dominant and creative type and sometimes other girls brought ideas and advanced. Sometimes I felt a part and sometimes a type of an observer from the side. It was not easy for me to bring my part in the project, but I felt that there is productive cooperation and there is the work of brainstorming, and we all invest the best of our talents and ideas. I learned that teamwork is cooperation between all the team members and is not just a part of the staff of workers and the rest are 'observers'."

The acquaintanceship from last year enabled the understanding and support of one another.

"It is natural that in a group of eight people there will be somebody whose voice is heard more and somebody whose voice is heard less. However, since among us there is prior familiarity from last year they did not let this happen and it was important to us that everybody make their voice heard and take part in the project."

Academic Seriousness and Relationship with Experts

From my observations, it was apparent that the general satisfaction and personal satisfaction of the students were felt as they showed greater expertise in the chosen mathematical contents. The learning and in-depth study of the mathematical skill through reading articles on the topic and engagement in activities that the textbooks offer caused a broad understanding of the difficulties innate in it and thinking about the teaching strategies suited to it, so that they would provide a response for all the students, including those with difficulties. In the Red Planet Group, in the Ratio Group, and in the Polygon Group it was interesting to see the way in which they processed the mathematical skill. In the first group meetings, each one of the group members brought an article that he found on the topic and read to everybody the sentences he had highlighted (emphasized) beforehand. The student who was in the role of the teacher concentrated the main points and at the end of every session summarized to them the important properties of quadrilaterals and aspects that they must take into account in the work on the project.

All the groups needed my mediation at different levels. The groups that less needed my mediation and guidance were those that had mastered the mathematical skill, or had achieved a good level of mastery in the three sessions, or the student who was in the role of the teacher in the same group functioned on a high level. These were helped by me in the group feedbacks or in the feedbacks in the assembly for aesthetic corrections or in other words, for technical direction or for mathematical accuracy.

The groups that needed me especially were the groups that had not mastered the mathematical topic and did not have a student-teacher with prominent leadership abilities. These groups received from me an initial lesson inculcating the mathematical skill, primarily on the topic of the geometric shapes and the circle. After the first lesson, in which I instilled in the members of the five specific groups (from the fourteen groups) the basic concept and the required skills and strategies, it appeared that the motivation in the group rose, as did the curiosity, the active inquiry, and the activity.

The members of the group of the application of the clock sought my advice in the field of arithmetic and turned to experts in the Gordon College of Education to the head of the department of innovation and technology in teaching and to the lecturer in the field of Hebrew language. The members of the Red Planet Group saw many films of origametry and tried to contact Miri Golan, an expert in the field of origametry, but they were very disappointed that all their attempts failed and she did not respond.

From the interviews and reflects of the students and their observations:

The reading of articles contributed to the increased depth in the topic and to the focus on the required mathematical skills and concepts.

"At the beginning of the process, it was decided that our project will engage in geometry. We went to the library and were helped by articles and textbooks and thus we became experts. Then we built a table, and in it we wrote the existing geometric shapes and geometric properties, so that in the process of the writing of the story we would not forget the properties."

The origametry group examined first the goals of the topic in the curriculum, and in the continuation each one of the group members summarized a different article and presented in the group and in parallel went in-depth in the learning of origami – paper folding.

"In order to build activity that will suit the students of the fifth grade and will challenge them, we performed an in-depth academic investigation – we learned the study material in the fifth grades according to the requirements of the Ministry of Education. In addition, we took many articles on the topic of geometry in general and origametry (geometry through paper folding) in particular. We learned how it is possible to prove geometric laws using paper folding according to the level of the material learned in the fifth grades. The articles were summarized by all the group members separately and then discussed them in the assembly. Every student presented the article he read, the main topics he discusses, the innovations that arose from the article that contribute to us to the activity that was chosen."

They were helped by the lecturers from the college, textbooks they wrote on the topic of geometry, and other textbooks.

"First, we searched for information, we entered the website of the Ministry of Education and all sorts of websites that include geometric information so as to further the knowledge on the topic and in addition to copy the axioms that are commensurate with each one of the shapes we chose to work with. Then, we examined the sentences with the book of Benny Goren and the book of Ilana Lavenberg so as to ascertain our understanding of the topic and deepen and extend our knowledge."

"First we examined the requirements of the Ministry of Education in the curriculum so as to see what are the topics that we focus on in the learning of the clock in the third grade. We examined in addition different sources of information, including articles, textbooks, reading books, and so on, so as to establish knowledge on the topic of the teaching of the clock and not only specialization in the clock itself (for instance, old clocks from ancient times and so on)."

Only after a number of rounds of feedback with me did they begin to think critically and to analyze every word in sentences they wrote for the geometric story.

"At the beginning of the work, it was difficult for me personally to think mathematically and critically, but after the first session with the lecturer, who opened my eyes to all sorts of nuances, my manner of thinking and the thinking of my teammates changed and became focused, we began to think about every sentence we wrote more critically, whether this is valid for all these shapes it is valid perhaps only for the shape in the story and thus it created in our story a high level of mathematical precision."

There were groups that only after the raising of the idea for the final product could go in-depth in the mathematical skill.

"The topic of the work did not arise first. Before it there arose all sorts of ideas for other topics from the members of the group. After advice with the lecturer, we decided on the topic of the polygons. To arrive at this topic, in a correct and in-depth manner, we were forced first to read about the polygons, to learn the topic, to specialize in it, and to be in full control so that we could build a good and mathematically correct presentation."

A student tells that her role was to summarize all the summaries of the group members and to create a review of the literature.

"My role was to summarize all the articles that everybody summarized, into one theoretical presentation that would be presented in the classroom."

The student tells about the difference in the work on the project from regular learning in which the teacher teaches in the traditional way:

"This is very different from traditional learning, in which the teacher says, here I am and open the book to this page. Here the student creates things from zero, goes to draw information from information sources, and seeks counsel exactly like we did in this project."

In this group I saw that the gap in the mathematical knowledge is large and the group members do not overcome it by themselves. Therefore, I decided to teach them by myself the way to discover the formula for the circumference of the circle and the area of the circle.

"In the first group session, we sat with the girls and thought in what direction to take this. In the end, after the lecturer taught us the ways in which the calculation of the perimeter of the circle and the area of the circle is taught we formed together an idea of a number of models that will illustrate the arrival at the formulas for the perimeter of the circle and the area of the circle."

The students who worked on the application of the clock were helped by lecturers in the college when I was not available in the field of language and computerized communication.

"Through the creation of the application we sought the counsel of different experts, including experts in the subjects of computerization, the Hebrew language, and experts in the topics of teaching the topics of arithmetic."

The digital book was written through a story that we wrote in the style of a story and rhyme. Their relationship with an expert in this stage was when they sought the advice of Dr. Revia Basis, a lecturer in the college on the topic of language. They sent her the story so that she would critique it and correct it in terms of language and syntax.

"Our relationship with the experts was primarily in all that is related to what is beyond the study topic. We sent what we wrote to the lecturer of language in order to be certain that we do not have language mistakes and we investigated the application with the head of the department of innovation and technology in teaching so that we can be experts ourselves in all that is related to this."

Sometimes an expert in the field, like in the field of origami, was not available.

"During the work on the project, we were surprised to discover that the field that combines between origami and geometry already exists in the field of teaching and is

even promoted by the Israeli Center for the Art of Origami. We tried to contact one of the developers of the program, Miri Golan, but we did not succeed. But we were greatly helped by information about the program found on the Internet website of the Israeli Center for the Art of Origami (http://www.origami.co.il). From this perspective, this was our relationship with experts. Additional experts we saw fit to consult are our accompanying teachers, we received from them many insights and ideas how to improve our project, such as to move among the pupils during the folding of the paper in the class and to ascertain that they folded correctly and discerned the geometric shape that we wanted them to notice."

Following my recommendation, one of the groups was assisted by an expert in the field of the preparation of the teaching aids – the instructor in the pedagogical center in the Gordon College of Education, who helped them in their work.

"During the work, we got messed up with the materials from which we could build the teaching aids kit, we understood from Miri that the pipe cleaners are not effective for us for the illustration of the quadrilaterals. She counseled us to approach the Pedagogical Center of the Gordon College of Education and to use the material called Polygal (polycarbonate) and dividing pins that can solve for us the problem of dynamism. We discovered the Pedagogical Center, at a relatively late stage, since we thought that we could do it on our won. We were helped by the instructor who is found there, who gave us nice ideas from her experience, and thus it was decided that the Polygal material would take the place of the pipe cleaners for the sides. In addition, she helped us with the idea about the measurement of the angles."

The Orchard Group was helped by a carpenter, a relative of one of the student, for the construction of the teaching aid that illustrates the different quadrilateral from wood.

"*We called the uncle of Dua, who is a carpenter and also her father, who is a professional. We explained to him what we want exactly and he understood, more or less … but he preferred that Dua show him exactly what we want, what we want … we decided to delineate on the computer the shapes and the idea itself and the parts, how this is supposed to look … then we sat when we are enthusiastic with the idea, we showed*

you, and we explained to you what we want to do … you very much liked the idea and approved for us to continue with it."

4. The Second Theme

The second theme is "Your approach, not your talent, will determine how far you reach." (Zigler). This is the empowerment of the student in active and involving learning through projects in mathematics – from fears, frustrations, and difficulties to curiosity, motivation, self-direction, and satisfaction.

In this part, I will conduct a discussion on each one of the indices that are mentioned in the theme 2 and that illustrate the socio-psychological atmosphere and the behavior of the students through my observations and through quotes from the reflections of the students, the interviews, and the observations of the observers in every group.

'Navigating in the Dark' – Concerns\Fears.

From my observations, towards the end of the first lesson, in the course on the development of initiatives and projects in mathematics, according to my planning and at the request of the students, I explained the course requirements with open and vague explanations. I said that they must work in teams as 'young startup founders' on the development of an original and innovative product for teachers of mathematics in the elementary school, so that every teacher who wants can take the product from the shelf.

In reflections in writing and orally, most of the students reported the feelings of ambiguity, confusion, concern, fear, pressure, curiosity, frustration, and even skepticism about the project success. Most began with many question marks about the continuation, some have a project that itself seemed difficult and complicated, and meeting timetables with the partners in the group (from a different culture as well, Jewish and Druse) for the creation of an optimal product only made things more complicated. Their recollections from the group work and the success in this work were not especially positive.

These feelings were apparent already during the introduction lectures, when I gave the general and open instruction regarding the final product that they must present at

the end of the course. In their words, they did not understand in-depth the rationale, the goal of the course, the direction, and the process. Some maintained that they felt that I am setting before them very high standards for the continuation.

The students who participated in a project-based learning course in the sciences in the previous year were also concerned. In the previous course, the students learned through projects and researched the process that the children experienced, while this time, in this course they had to work themselves on the project as active, independent, and involved learners. I believe that through self-experience they can learn better, and therefore I changed the initial format of the course I initiated in which the students taught children around the projects.

From the reflections, interviews, and observations of the students

The unknown path, sometimes with unfamiliar students, and the requirement to initiate something innovative in mathematics caused many fears among the students, the asking of questions, and sometimes frustration. Over the time the picture became clearer.

"At the beginning of the course, in the first session the lecturer asked – to create a project, something that doesn't exist in the market, a new product that does not exist and we will develop, we need to be a type of entrepreneur. Another concern was that I did not know anybody beforehand in this class, and I asked myself how do I go to prepare a project of such a scope with people whom I do not know? In what direction will I choose? Who will be my group members? And more … slowly the picture became clear, we created for ourselves a cohesive group and it should be noted that the composition of the members in the group was especially heterogeneous. We are different one from the other, but this difference and the definitions that we defined for ourselves as a result in this work prepared us well for what comes."

The data are from my observations and from the reflections, interviews, and observations of the students.

In addition, I asked the group members to rank the reasons for the fears from 1 to 3.

Table Number 2: Reasons for Fears

Groups	Fear about open guidance – of Going into the 'Unknown'	Fear about the many members in the group (or heterogeneity)	Fear about the differences of opinion in the group	Fear about the lack of meaning in the group	Fear about the lack of creativity	Fear about the lack of success in building the product from a technical perspective	Fear of lack of mastery of mathematical skill
Ratio Clips	1	2	2	2	1	2	2
Quadrilateral Family	1	1	1	1	1	2	2
Orchard	2	1	1	1	1	2	1
Angle in the Clock	3	1	1	1	1	2	1
Multiples of 7 and 8	2	1	1	1	1	2	1
Monopoly of Three Dimensional Objects	3	1	1	1	3	3	3
Fibonacci in the Olive Grove	3	2	2	2	1	2	2
The Exploding Box	3	2	2	2	1	2	1
The Clock Application	1	2	2	2	1	2	1
The Red Planet	1	1	1	1	1	2	1
Gil and the Circles	1	1	1	1	1	2	1
Revolving Three Dimensional Objects	2	1	1	1	1	2	2
Kingdom of Polygons	1	1	1	1	1	2	2
Teaching Kit for the Circle	2	1	1	1	1	2	2

Key:
1 – Low level of fear
2 – Moderate level of fear
3 – High level of fear

According to most of them, only in the group sessions that I joined did the picture begin to become clear, and only in this stage did they feel calmer and therefore more practical. The stress and the anxieties about the unknown vanished, and from this point onwards there began conversations, deliberations, and choices.

"At the start of the semester, in the introduction lessons, I was very afraid of the uncertainty and I feared what was going to happen during the course. Every one of the members of the group did not really understand what her role is and what she is supposed to do. It is possible to say that 'we navigated in the dark'. While we had a topic, we did not know what we need to do with it. It was hard for us to disconnect from the 'traditional' and obvious thought that the lecturer needs to tell us what to do and then we go and do it. This is a problem that indeed can appear also among the students who are not accustomed to this form of learning. It is hard to change this method of work, and therefore I felt a sense of frustration."

They felt as if they were thrown into the water without being ready for this.

"In addition, it is important to note the fact that we were thrown into the water alone, without a definition ahead of time what we must do. The pressure is tremendous and caused us to make many changes in the process, even in the earliest stages."

However, there were also individual students who felt comfortable with the introduction lectures and with the open guidance. One of them stated that:

"Me, personally, I do not fear uncertainty – since I believe that in the end everything will work out, everything has a goal. Therefore I was not alarmed directly like most of the girls, and I understood that I will understand the nature of the project soon. In addition, I know the lecturer from previous courses and I know that I will derive the utmost from such a course."

The groups that feared the 'going into the unknown' on a moderate level were the Orchard Group and the Multiples of 7 and 8 Group, the group of revolving three dimensional objects, and the group of the circle kit.

The Orchard Group was heterogeneous, half Druse and half Jewish, and the Multiples of 7 and 8 Group was primarily new immigrants from Russia. Therefore, it is possible to associate their fear also to the difficulty with the Hebrew language. In addition, they were afraid about their lack of technical abilities for the building of the product. In the Circle and Revolving Three Dimensional Objects Groups, it was clear that

the fear revolved around the lack of mastery of the mathematical skills and the technical abilities required for preparing the product.

In the Ratio Films, Exploding Box, Fibonacci Sequence, and Clock Application Groups, the fear was primarily about the many members of the group (seven). They were afraid that there would be no discipline in the group, that there would be many differences of opinion, and that they would need to 'chase one another'. In the Fibonacci group, there was another fear of heterogeneity, since two students were Druse and the rest were Jewish. One of the Jewish students expressed her fears from work in such a heterogeneous group because of the Druse students' lack of mastery of the Hebrew language and because of the distance from their homes that would make the group meetings difficult. Rather quickly they saw that the meetings occur during the time of the lessons and that the communication between them is at its best.

In the Red Planet Group, the origametry, there was no fear about the many participants although there were seven members. Apparently there was a student who was the leader in the group and who conveyed confidence and calm. The student who led the group already from its initial stages served in the past during her military service as an office and was chosen in this course by her group members to be the 'teacher' in the group.

In essence, every person who was afraid of the many members in the group was concerned primarily about the differences of opinion expected in the group, the expected lack of discipline, the idle chatter and the laughter, which would take precious time, the differences of culture (Druse and Jewish), and lack of concentration that would be the result of all these.

Some feared the work in a large group, primarily after unsuccessful previous experience in a large group.

"The very fact that I was a part of a large group really caused me to feel pressure and I thought how could I work with a large group, especially when in another course last year we did a work together, four girls, and this was not successful in my opinion."

"At the beginning of the course, when I discovered that the work in the course is group work that includes a large number of students together I was very scared. Joint work of a number of girls together is daunting at the beginning, and there are many fears that arise, such as, for example, how are decisions made? Will there be understanding among us? Can we form an opinion that will be agreed upon by all together? And many other worries that influenced me at the start of the path."

Beyond the fear of frictions, the students feared that they could not express themselves appropriately in a large group.

"During the project that we did together, the entire group, first fears on my part arose – how it will be to work in a group with so many girls, so many different opinions? Will each one of the girls receive the stage and the freedom to tell what she does or thinks? Will this be authentic? Will each one be expressed in the joint work? (Of course, every one of my many fears and question marks received answers and I see many advantages in work through a group and also believe that through group work it is possible to derive considerable benefit and learn much from others.)"

In the Monopoly of Three Dimensional Objects Group, there was a most significant delay in terms of the progress at work. All the members of the group were Druse and Christian Arabs. They were very fearful about the open task, because of the work in the Hebrew language, the lack of control of the mathematical skills they chose – geometric shapes, the creativity, and the technical abilities they must display. All these caused them to be stuck for a number of lessons. They were able to continue onwards after I dedicated to them a private lesson on the topic of the shapes in general, including an in-depth understanding of their properties. Immediately they felt experts in the topic and began to think and to plan the product.

Frustrations and Difficulties of Students

From my observations, many students felt a certain frustration because of the open and vague guidance that I gave for the work on the project and in the lessons after it. Sometimes they were disappointed when they understood that a certain idea cannot

grow into a meaningful product. It is possible that my considerable experience in mathematics in the elementary school as a teacher and instructor of students and teachers caused me to think that I know more than others what can and what cannot succeed as a project and perhaps thus I slightly held them and did not allow them absolute freedom of choice. In any event, it would not be correct to criticize myself without examining my motives in one or another specific situation. Indeed, when I returned the conversations and personal feedback in the groups I understood that I enabled the students during a number of sessions to think, to search, to read, and to talk among themselves about their idea and only when I saw that they are not 'taking off' and going anywhere did I gently hint that they should think about a new path.

The lack of knowledge in the mathematical skill constituted a difficulty in half of the groups. The students understood that they must learn the mathematical topic in-depth, on the level of the teacher, and deliberated about the mathematical skills and the concepts that they had to focus on. Some felt frustration since they could not see in the first stage in front of them the final product. In addition, the task obligated them to become independent learners with self-orientation, to read articles on the mathematical topic they chose, to dig and solve in the textbooks, and all this in cooperation with the teammates and during the lessons. In some of the groups, greater gaps were apparent in knowledge, in the groups of shapes, the circle, the ratio, the exploding box, I mediated and taught the main mathematical skill in a personal meeting when I saw that their motivation declines because of these gaps. Gaps in knowledge caused them in the process of the building of the product to repeat their way and to correct, sometimes with mental aggravation. Thus the students from the Ratio Group corrected themselves when they discovered a mathematical error or an error in the name of the number in the film they filmed. They filmed again, although this process took from them sometimes the afternoon recess in the college. In addition, prominent frustration was felt among the students of the project of the polygons when in one of the feedbacks together they were given criticism on the rhymes in the story they wrote at the expense of mathematical accuracy.

In all the groups there were sometimes differences of opinion; however, the student who was in the role of the teacher and who was chosen with the agreement of the

group members reassured each time and straightened things up. Sometimes the student who is the 'teacher' attributed to herself greater control than necessary and thus caused temporary passiveness among the other group members, as happened in the Fibonacci group.

In all the groups, technical difficulties were felt, such as, for example, the use of an inappropriate accessory such as pipe cleaners to prepare polygon sides, bugs in the application, failure to find rhymes in the story, or finding rhymes at the expense of the mathematical accuracy, filming a film again, too little time in the course lessons which requires additional meetings at the expense of the free time, or corresponded via email or WhatsApp.

A large number of group members in the project constituted additional difficulty, since sometimes there were uncontrollable laughter and small talk unrelated to the work, differences of opinion, and arguments.

Another difficulty was the lack of mastery of the Hebrew language. Some of the Druse or Christian Arab students speak Arabic as a mother tongue, and two students had immigrated to Israel from Russia two years before and still had difficulties with the Hebrew language.

From the reflections, interviews, and observations of the students:

My empathy towards them was important and strengthened them throughout this process, even when they were frustrated about not finding an idea for a product or about criticism they did not like.

"We thought about the search for strategies on the topic of vertical multiplication. In addition, we had three articles that are related to the topic, but we were afraid and we felt lack of confidence and lack of certainty. We were not sure that this is the direction of the project. On the other hand, we hoped that our choice would suit the goal of the course and this definitely would cause us comfort. We entered the meeting with Miri, and we presented the idea to her. She answered us with a smile, understanding, and empathy that the topic of vertical multiplication is a very limited

topic, technical, and slightly boring. Miri explained to us the way for the choosing of the idea, she suggested to us a number of proposals, including a project on the topic of the perimeter of the circle and the area of the circle. When we left the room, there was a feeling of disappointment since we had hoped that vertical multiplication would meet the requirements but the idea was apparently not successful."

"I did not succeed in knowing and understanding what exactly is required of me, and what it is to develop a project in mathematics, especially when I come to the program of retraining for mathematics, which is an area that I did not study during the undergraduate degree. This fact made me more pressured, when I am disconnected from the field of mathematics from the period of the school and now I am required to develop a new project that was not proposed previously. This caused me a feeling of confusion, pressure, and frustration."

In the beginning, the frustration was from the lack of the understanding of the task.

"At the beginning of the course, the primary difficulty was to understand the requirement of us, what exactly we must do so as to make the work better."

When the members of a certain group saw that there are others in other groups who are frustrated, they calmed down a bit.

"Our first two sessions were filled of frustration and pressure of each one of the girls, which led to 'disputes' and arguments with one another. We attempted to hear what the other groups do, whether they understand, and we felt that we are all working and only we are engaged in understanding what is happening here. Slowly we discovered that other groups do not understand what to do, and this was a little encouraging. From session to session, the situation improved and we felt that we are beginning truly to advance although there still was a feeling in the air that we are working without understanding about what and to where this is supposed to lead."

To overcome the frustration because of the gap in the mathematical knowledge they decided to go in-depth into the traits of quadrilaterals.

"*We decided that before we advance in the work, we need to master well the properties of the quadrilaterals and the relations of inclusions. In this meeting we went over all the properties of the quadrilaterals."*

In one of the feedbacks in the plenum, one of the groups received unpleasant criticism from a number of students on the lack of mathematical accuracy in the rhymes they composed for the story about the quadrilaterals.

"A difficulty we encountered during the semester was in the first showing of the presentation that we built before the entire class. We came to the presentation ready, confident in ourselves, filled with enthusiasm from what we did and our product. Shortly after the beginning of the presentation, we received very unpleasant criticism from the classmates (while the criticism was correct, it was very difficult to hear it). This dropped on the group members difficulty with continuing and frustration. What lifted us up to continue to work and to correct according to the comments with full strength was the instructor's mediation, the group cohesion, and the commitment of the girls in the group to one another. In the end, we built a very invested project, intelligent, creative. The final product – the presentation, is very colorful, rhymes, and has entertaining characters that the pupils will be curious to know."

"Throughout the work, there was one time that we were asked to present before everybody, in front of the class, our situation at work. We came with great confidence for the presentation, but the criticism we received in the assembly was not so pleasant and was even difficult to hear for us. This slightly decreased the group confidence, but among the members of the group there was great support and it was said that we won't let this influence us and we will attempt to understand what they intend and to draw conclusions. The criticism we received in front of everyone was something that in retrospect helped us make our work better than it was in the beginning, in terms of the mathematical language becoming professional and correct."

Difficulties also derived from the need for critical thinking, to which they were not accustomed.

"At the start of the work it was hard for me personally to think critically and mathematically, but after the first session with the lecturer, who opened my eyes to all sorts of nuances, my manner of thinking and the thinking of my teammates changed and became focused, we began to think about every sentence we wrote more critically, whether this is valid for all these shapes it is valid perhaps only for the shape in the story and thus it created in our story a high level of mathematical precision more than there was before the lecturer's focusing.

The members of the group described here went in-depth on the topic of the decimal structure but did not find an idea for a product for a number of meetings except with my mediation.

"We chose the topic of the 'decimal structure', and during the lessons we worked on articles that engage in this topic alone. We investigated it, and we learned it in a thorough manner, through books in the library and different Internet sources. This, until we reached the idea towards the end of the semester for the final product. When we sought counsel with the lecturer, we discovered that our original idea is interesting and creative, but that to do only on the decimal structure is perhaps not enough. We thought together with the lecturer where we would take this idea, and then we formed together an opinion. We chose to build a model of the 'exploding box' that includes navigation cards in mathematics for the fourth grade pupils in order to accompany them during the year. After the idea was raised, all the preparations for the product gained momentum very quickly.

A student who is not accustomed to teamwork sought to work alone. When she understood that this is not the requirement, she continued to foster antagonism and then attempted to take control. Only after all the members of the group agreed with her idea did she calm down. The atmosphere in the group improved from session to session.

"*At first I felt slightly anxious and bothered when the task was presented, the reason was that I did not know anybody in the class and this method of work in groups was 'strange' for me. At first I behaved in a slightly defense manner, I said that I will do this task alone, I will present the project by myself, and I do not have a need for a group.*

Then I was told that I do not have the option of performing the project alone, I looked at the class angrily and I saw a group that is not decided on its topic – and this caused me to 'leap' at the opportunity given to me. I joined the group and I said, 'I have a wonderful idea! Let's base our project on the Fibonacci sequence!' I encountered great objection when I suggested this idea, but I did not stop attempting to 'pressure' the group that this is an easy and fun topic for presentation. After the group agreed with me on this, we went on the way."

"Chen wanted to integrate the work with art, I wanted more a tool that can be used by the students for investigation and a tool that will serve us as teachers for illustration, eventually we combined the topics."

There were disagreements but in all the cases they reached an agreement from seriousness and responsibility and commitment to the project.

"I was given the possibility of seeing how we conduct a productive discussion, how each one has her place and the possibility of expressing a critical opinion about what the other says, and this without the intention to harm but to lead to places of additional critical thinking, we did not always agree on all things, and sometimes there were differences of opinion, but in the end we came to an agreement through seriousness and full responsibility from all the girls in the group."

Sometimes when there were disagreements the student in the role of the teacher mediated and created a situation of agreement.

There were disagreements but in all the cases they reached an agreement from seriousness and responsibility and commitment to the project.

"Elinor, who was in the role of the teacher, took the role with full responsibility and seriousness and directed us frequently when there were different arguments, and thus we decided together what we are doing next in each and every session."

There were also technical difficulties, not everybody had an iPod, not everybody knew how to use it.

"In the beginning, there was the concern since not everybody has an iPad and not everybody knows to use one but after further thinking it was decided finally that our product will be accompanied by an application. We all recruited to know and recognize how it works."

Difficulties in finding suitable materials were resolved using conversations between the group members and with the lecturer.

"After making the decisions and beginning the work different technical difficulties were created, in which the difficulty of preparing the product, finding a creative solution using discussion with all the members of the group and the lecturer."

The building of the model was not simple, since accuracy and completeness were important.

"Building the models was my role. I will not lie, the building of the models was complicated, not easy, and took a lot of time. I made certain that everything would be as precise as possible and that no item would be missing. In addition, I made certain that everything will be colorful and large so that truly I could implement and use the model also in the continuation of the path as a teacher."

A large number of group members in the project constituted additional difficulty, since sometimes there were uncontrollable laughter and small talk unrelated to the work, differences of opinion, and arguments. In the written reflections and in the interviews it was apparent that the students agree that there needs to be a maximum of four or five members in the group. Some recommended that the role of the observer needs to move in rotation so that everybody will feel the atmosphere from the side and will become more responsible.

"After two lessons, everything became clear, and I understood that we must divide into groups. Very quickly we chose the group and after we looked again we saw that we are a very large group, about seven girls, which made me a little pressured, and I thought how we can work so that each one of us will express her opinions. I received the role of observer. I had to observe the members of the group and document their feelings,

illustrate the processes that occur between us. It was not so understood how to do this, I never was in such a situation when I needed to write how my friends feel and especially when I do not know them well."

To avoid unpleasant disruptions, fits, and friction, they thought to change the observer in a rotation so that everybody could look and feel the atmosphere from the side.

"I would change the responsibility of one observer and let a number of students observe so that they would feel a part of the project and will be responsible. In addition, I would limit the number of students in the group into five groups of students although we got along in a group of eight girls. I think that five could have been far easier."

This quote describes feelings of a student who felt passive in his groups.

"To the best of my understanding, in the work in the group the process of the work with the girls, not everybody had the same place and the same verbal and technological abilities and it was not always easy to express them but this was logical and reasonable for this to happen since in the large group dynamics that were not always possible to anticipate are created, sometimes somebody took the initiative and leadership, for instance, Yamit, as a very dominant and creative type and sometimes other girls brought ideas and advanced. Sometimes I felt a part and sometimes a type of an observer from the side. It was not easy for me to bring my part in the project, but I felt that there is productive cooperation and there is the work of brainstorming, and we all invest the best of our talents and ideas."

Language difficulties are described in the following quote:

"Although the mathematical language is universal, I had difficulties and fears about the Hebrew language since I immigrated to Israel only two years ago. Now, after I have completed the project, I can say that the work was very good for me. Primarily in terms of my development, I got to know new concepts, I acquired new knowledge, independent experience."

Student Curiosity

From my observations, the members of the group who were not especially afraid of the open and vague guidance displayed considerable curiosity regarding the process they experienced, the mathematical skill, and the expected product.

This curiosity is what motivated them to the choice of the mathematical skill, learning it in-depth, and organization and planning of the product until it is formed. Students who were especially afraid of the autonomy they received displayed lack of curiosity in the early stages, lack of motivation to work, and even lack of belief in their self-efficacy and a certain sense of being stuck because of it.

The curiosity derived primarily from the desire to see what the final product would look like.

"*The performance of the project inspired in me considerable curiosity since I was interested in what the final product would look like."*

The students felt curiosity about the process when they decided to connect between the Fibonacci sequence and the topic of the raising of olive trees.

"*The first meeting, which included a limited composition of the members of the group, included in the end also a meeting with the instructor who gave feedback and additional explanation on the process of the project. On the level of the group, the topic of the Fibonacci sequence interested all the group members and awakened their curiosity, and it was apparent that each one wants to contribute and to help. Every session began with brainstorming when during it we talked about the chosen topic. How should it be presented? What products do we want to obtain at the end of the project? Naturally, such ideas and others were raised and after a discussion were left on the floor of the 'editing room'.*

Student Motivation

From my observations, about one half of the students reported feelings of ambiguity and being stuck in the first lessons of the course, which are accompanied by

many concerns and doubts regarding the success of the project. Prominent images of their feelings were “walking into the unknown” and “navigating in the dark”. All these caused them many inhibitions in the first sessions and lack of motivation to advance.

Others were not alarmed by the guidance. Rather, the very fact that the guidance was open caused them to evince curiosity, interest, and motivation to begin to work on the project.

The members of the Kingdom of Polygons Group reported a temporary decrease in their self and group confidence and in their motivation to continue to work, when in one of the lessons they were asked to present a progress report of their project in front of everyone and they received feedback from their fellows that was critical, merciless, and even traumatic in their eyes. They wrote a lovely story in rhyme and with humor that addresses the “Kingdom of Polygons” and the “Princess Parallelogram”. However, the course members identified inaccuracies in the presentation of the attributes of polygons, which derived from the desire of the group members to ensure rhymes and humor. The lessons were divided so that in some of the lessons there were progress presentations and feedback in front of everyone and in other lessons there were personal meetings with me or without me, so immediately at the end of their progress presentation and the feedback, which was traumatic according to them, I met with the frustrated group members who saw that all their efforts were going down the drain. I felt their drop in motivation to continue to work, but when I showed them that there was no need to write the story anew they immediately perked up. They saw that very easily it is possible to find alternative and accurate sentences and to minimally correct the story so that it will continue to be fascinating, rhyming, and primarily accurate in geometric terms. After only ten minutes of slight changes, they began to see that it is possible to correct the few inaccuracies easily and effectively, while preserving accuracy, rhyme, and humor. Their enthusiasm after the corrections that they found themselves was tremendous, and it motivated them to continue with energy.

My additional interpretation following the table that addresses the reasons for motivation in the continuation of the chapter:

From the reflections, interviews, and observations of the students:

Here a student describes a decline in the motivation following the feedback I described.

"*One of the times that we were asked to present in front of everyone our progress in the work, we came with considerable confidence and pride in our forming creation, but the criticism we received from our peers was not very pleasant and even hard for me to hear. This slightly reduced the group confidence, but among the group members there was a high level of support. We said that we would not let this influence us and that we would attempt to understand their intention and to draw lessons. In retrospect, the criticism we received in front of everyone helped us make our work better, mathematically correct, and more professional than it was in the beginning. From that moment, we began to think more critically in mathematical terms. We sought the help of the lecturer who shed light on our issue for all sorts of nuances and helped us find appropriate ways."*

Another member from the same groups said: "*I must note that when we were asked to present the product in front of the class, we came with a good feeling, that we have in our hands a good product, and that we would receive good criticism from the other students. After a number of minutes, our presentation was stopped and we received harsh criticism from the course members, not like we expected. This criticism caused me personally a difficult feeling and a decrease in confidence. The girls in the group had a feeling similar to mine but we strengthened one another and did not let the criticism 'get us down'."*

This case is a single case of one group from fourteen groups in which a sharp decline in the motivation to continue to work was seen. However, the group members recovered rather immediately and saw that the situation was not so terrible. Two other groups appeared to lack motivation in the initial stages since all the mathematical skills they raised were rather quickly disqualified, such as vertical multiplication or addition and subtraction with conversion. These skills were learned in a technical manner with the understanding of the decimal structure and with the accompaniment of arithmetic stories.

However, all thinking regarding the project on these topics did not succeed. However, many reasons for an increase of the students' motivation to work on the project were found, when the most common ones were collaboration in the group and the moment in which they had the idea for the final product.

All the groups, without exception, reported a wonderful friendship that occurred already at the start of the work. Some indicated that they would remain friends in the future, and they would even do future work in the same group composition. The friendship strengthened and encouraged them also during ambiguous and not simple moments and preserved a high motivation to work.

"I saw that the cultural difference between us worked well and in our favor despite my fears. When there are attention, shared learning, and brainstorming, the result is good products! Each one has her place and her say and this caused me to have very great internal motivation and great drive to success in the project. I felt that I have somebody to work with and cooperate with. It is clear to me that if the situation were different we would not succeed in creating as good a product as we created."

The enjoyment of the collaboration caused motivation in the group that built the ratio films.

"*During our meetings, I realized that all the students come, all come on time, all bring what they were required to bring and do not forget, behave in an adult manner and are respectful of one another, taking advice from one another, speaking appropriately and respectfully, it can really be said that we have drawn close and learned to know one another better. The work in the group was enjoyable and fun, all the students contributed to the work, did what was assigned to them. I was surprised that truly everybody cared and everybody comes to the sessions as if this is a lesson with a lecturer and this is something that is compulsory but since everybody truly cares and it is important that everybody came and we worked in exceptional cooperation."*

Two groups felt lack of motivation until they reached the idea of the final product. Then they began to read articles and textbooks on the mathematical skill, to counsel, to organize, to divide roles, to plan, and to create.

"We decided that we are changing and switching the topic to the topic of the circle, which is considered a more complex and difficult topic in geometry for sixth grade pupils. Immediately after the change of the topic, we began with the collection of theoretical materials – articles on the topic, guides for the teacher, and textbooks for pupils and we began to think what we want to achieve from the project, how we want this to be built and we began to build sketches."

This quote describes how from meeting to meeting and from feedback to feedback the product is created and with it the motivation increases.

"As the sessions advanced, we saw how a small idea becomes reality and how the thought that we had in our head is created in front of our eyes. In every session, we received feedback from the lecturer, and according to her comments we corrected and improved the presentation."

"When an idea is created", like the connection of mathematics to science, the motivation is at a highest point:

"From the moment of the choice of the idea I felt that our motivation is increasing and our desire is growing to advance forwards in initiative and to continue to develop the idea. The enjoyment is to work in a group, and to achieve our goal was enjoyable."

"I felt that a wonderful feeling and motivation to continue the moment we succeeded in connecting between mathematics and sciences, between a Fibonacci sequence and the increase of the olives in the groves, which is a part of my life. My husband and I live and breathe olives all year and care well for our groves."

In another group, the motivation to work rose when they decided to connect between geometry and origami. A little regression occurred when they decided to turn to

Mrs. Golan, the founder of the Israeli Center for the Art of Origametry[5] in Israel, and did not receive a response. In any event, they immediately recovered because of one of the members of the group functioned as a teacher of the group and promoted the work energetically.

"*To examine the relevance of our project for the pupils of the elementary school, in the studies of mathematics, it was decided to have an 'experiment of tools': two attempted to convey the presentation in one of their lessons in their experience class in the school. We held a discussion how it is necessary to have the lesson itself, whether to begin with the building of the origami and then to go back and show the geometric shapes that were created or when during the building of the origami shape it is necessary to present the shapes created and to discuss them. There was success and enjoyment and this shaped the motivation to upgrade the story and the origametry work."*

The motivation of many students derived from their desire to understand themselves how children learn through projects. In other courses of PBL, the students teach children through projects, and in this course they experience themselves. In this way, they can understand exactly the feelings of their pupils in the future.

"Now I can understand all the future pupils when I will teach mixed and active learning. I learned much from you what should be revealed and what should remain to the students to discover themselves. I learned that you should not be upset from question marks and if you are patient and wait then the picture will become clear, the thinking of the pupils is not set in stone and they are allowed to take off and create products that previously it was not thought that they could perform. In addition, I learned how to flow with the ideas of the students and not to dictate to them but to lead them to invent ideas of their own and only to help them improve them."

Among others the motivation rose when they received positive feedback in front of everyone or when it was discovered that there are many theoretical contents on the topic they chose, articles, textbooks, and Internet websites or when it came to their mind

[5] Origametry is the folding of paper in the learning of geometry

an idea that nobody else had formulated before them and that were agreed upon by all or when they succeeded in writing a story in rhymes after many difficulties.

"When we found a way how to link the rhymes and the word we wrote into the story suddenly it began to appear really like a magical and lovely fable. I took the story, and I put it into the presentation. I began to play with the characters in the story and suddenly the fable began to be really true. In this stage I already was enthusiastic with our final work and I understood that truly we have created something of ours and that it is really nice and special. I showed other friends the presentation and they were enthusiastic and it was exciting to see that, after all the fears that I had at the beginning, in the end something wonderful was the result."

The members of the Orchard Group were proud of the idea that they had, and this motivated them to work energetically. They planned in the continuation to register the product as a patent.

"We chose my idea that they liked and they saw it as a product that I can receive a patent for it, and yes, I am not in the stage of the patent registration. It can be said that if there had not been group work and there was no cooperation as there was in our group I think that I will have difficulties to complete or perform the work in a good way as we performed it."

The reasons for the motivation of the students, from the written reflections, the interviews, and observations, both theirs and mine, are presented in the following table.

Table Number 3: Reasons for Motivation

Groups	The need for self-experience in PBL	Exciting idea that arose and/or accessible materials for mathematical skill	Mediation of the lecturer	Becoming experts in mathematical skill	Pride in the forming work	Positive feedback in front of everyone	Collaboration in the group and/or friendship
Ratio clips	3	3	3	3	3	3	3
Quadrilateral family	3	3	3	2	3	3	3
Orchard	3	3	3	2	3	3	3
Angle in the Clock	3	3	3	3	3	3	3
Multiples of 7 and 8	3	3	3	2	3	3	3
Monopoly of Three Dimensional Objects	3	3	3	3	3	3	3
Fibonacci in the Olive Grove	3	3	3	2	3	3	3
The Exploding Box	3	3	3	3	3	3	3
The Clock Application	3	3	3	2	3	3	3
The Red Planet	3	3	3	3	3	3	3
Gil and the Circles	3	3	3	2	3	3	3
Revolving Three Dimensional Objects	3	3	3	3	3	3	3
Kingdom of Polygons	3	3	3	2	3	3	3
Teaching Kit for the circle	3	3	3	3	3	3	3

* The table was created following their reflections, observations, and interviews and following my observations.

In addition, I asked the group members to rank the reasons for the motivation from 1 to 3:

Legend:
1 – Motivation on a low level
2 – Motivation on a moderate level
3 – Motivation on a high level

From the table it is possible to see that the motivation of the students at a high level derived from all the reasons noted, with the exception of their transformation into experts in the mathematical skill. In the Quadrilateral Family, Orchard, Multiples of 7 and 8, Fibonacci, Clock Application, Gil and the Circles, and the Polygon Kingdom Groups, we see that the motivation increased to a moderate level since the students showed reasonable expertise in the mathematical skills and they had to enrich themselves only slightly. Their motivation was rather immediate and not like the other groups whose motivation only came after they became experts in the mathematical skill.

In contrast, the Ratio, Angles of the Clock, Monopoly of Three Dimensional Objects, Exploding Box, Red Planet, Revolving Three Dimensional Objects, and Circle Kit Groups, which had not mastered the mathematical skill, felt gaps in knowledge and understanding and felt they were stuck at the start of the process. Their high motivation came only when they became experts in the mathematical skill after reading articles on the topic, clarifying the activities in the textbooks, and mediation or instruction by me.

Self-Orientation in the Mirror of Autonomy of students

From my observations, the autonomy given to all the students in the different groups in the work on the mathematical projects reflected for them the self-orientation of each one of them. However, consistent progress in the work was seen only in the groups in which there was a student with leadership abilities and self-orientation. Five groups from fourteen groups delayed over the choice of mathematical skill or the choice of the final product. In some there were arguments and feelings of frustration and pressure, and some even delayed for half a semester, with explorations, deliberations, and back-tracking.

When the students were given autonomy and proof of their self-efficacy and self-orientation was required, it appeared that they held onto their friends as their saviors. I enabled the utmost autonomy and freedom in the choice of the mathematical skills, choice of the idea for the product and the composition of the groups, so that in the third lesson they could move in the classroom freely, talk with their friends, and connect to the groups. This freedom had decisive impact on the composition of the groups. All the

students who knew friends from previous courses or from the past connected together into the same group, regardless of the interest they displayed in any certain mathematical skill.

The grouping with known friends and the display of empathy, consideration, praise, and attention to one another made it easier for the students to accept autonomy, but it appeared that my mediation in the group sessions was critical in certain groups. In some groups, no prominent leader was seen, and the group members appeared passive and possessed of less self-orientation for independent learning, so that I met with them more so as to incentivize and motivate them.

From the reflections, interviews, and observations of the students:

The autonomy given to them rather silenced some of the groups. They did not know what they had to do since they were accustomed to explicit directives and not to open tasks. Only from session to session when the picture became clear did they begin to act.

"Our first two sessions were full of frustration and pressure of every one of the girls and this created 'debates', our arguments with one another. We tried to hear what the other groups are doing, if they understand and we felt that we are all working and only we are bothering to understand what is happening here. Slowly we discovered that other groups do not understand what they are doing and this is a little encouraging, from session to session the situation improved and we felt that we are beginning to truly advance although the feeling was that we are working without understanding on what and where it is supposed to lead."

Even when there was an idea, it was very hard to think how to advance with it independently.

"In the beginning of the process we chose very quickly the topic we would like to work on but it was very hard for us to think where we want to take it and it took about half a semester until we understood that we want to make a box game – game board and upon it to establish the chosen topic – shapes."

The autonomy given to them created many arguments until decisions were made.

"After deliberations we decided to build a game since a game allows the students to learn in an experiential manner."

When the students understood that they are given the autonomy to work, they connected with known people in the creation of the groups. However, they thought that heterogeneity is important for such work since each one contributes from his abilities.

"At the beginning of the course you announced that it is necessary to divide into groups, and I chose to go to the safest place, to perform the project with girls I know. The group was joined by another student whom I did not know. All the girls in the group were different from one another, and therefore each one contributed something different, 'other', to the project. In my opinion, heterogeneity is very important in the group and does not constitute a difficulty but adds only good."

Only from session to session did they understand that they are required to initiate and act by themselves.

"With the progress of the sessions, I understood from time to time through the orientation of the lecturer what is required of us and that we must build a 'new' project based on the learning material, through which we can teach our pupils in the future."

The good atmosphere in the group, the understanding, the empathy, and the consideration as well as the feedback from me constituted factors that strengthened and incentivized them for the continuation of the way.

"In every session, we received feedback from the lecturer, and according to her comments we fixed and improved the presentation. In the second meeting on the task, we already felt in the air the feeling of empathy that was created between the members of the group, what contributed greatly to the good teamwork and proper and pleasant conduct throughout all the work. For instance, one of the girls said a rhyme that she liked and somebody else less liked it or did not understand it and then we took the rhyme as a skeleton and changed it to a level that everybody agrees with and likes the product. It

was very important to use that everybody will be satisfied and will be proud at the end of the work about the project we produced."

In her opinion, there was great value to such work in terms of creativity, freedom, expression of abilities, and collaboration, and what made it easy was the fact that I, the lecturer, was always in the background.

"The meetings of the members of the group throughout the semester were filled with creativity, knowledge, freedom to express the abilities of each and every one of us, these were things that were not obvious and contributed to our work, to work with an open mind, without pressure, and without guidelines that would fix us in a certain thinking for the work. However, as necessary we always could to turn to the lecturer to seek advice, to ask questions, to make with her certain decisions. The work method in this task helped me personally develop my creative side (which does not come easily to me) and of course connected me to the girls in the group."

As I noted, in the groups where considerable motivation for work is seen and the self-orientation of the participants in it, there was a prominent leader who served in the role of the student - the teacher. Her leadership motivated the group members, united them, and contributed to the display of their self-orientation and the sense of flow. They made a relatively rapid decision on the issue of the mathematical skill, began to go into depth in it, and in parallel searched for an idea for the final product.

"The group where I worked formed relatively rapidly, we met in the morning of that day and there was a connection that it is slightly hard to explain in words. We understood one another and the orientation of each one, and there was flow in all the different areas of work. The communication in the group was high, every time that one of the girls raised an idea directly there was somebody who continued and understood the mood and the knowledge how and what to do from this point at which the member of the previous group. We helped one another and we tried to have the division of work be equal among all of us as much as possible, so that the work would not fall only on one or two students. We met every Sunday in one of the classrooms for nearly four hours each time, and the work simply flowed. More than once we spoke in a group that was intended

for such work, and each time one of the group members contributed further of her personal time to add to the work. The work was productive and fruitful, primarily in light of the fact that I greatly enjoyed it and this although the field of geometry does not come to me naturally."

The autonomy is expressed already in the initial stage of the search for the learning materials and for the in-depth study of the mathematical skill.

"The topic is what we chose after bringing up all sorts of different topics of the members of the group, when we had already chosen the topic and received the approval of the lecturer to continue with it. First, we began to search for information, we went to the website of the Ministry of Education and all sorts of websites that contain geometric information so as to further the knowledge on the topic and in addition to copy the sentences suitable to each one of the shapes we chose to work with. Then we examined the sentences in comparison with the book of Benny Goren and the book of Ilana Lavenberg so as to ascertain our understanding of the topic and deepen and extend our knowledge."

However, the members of the groups for which the autonomy was strange enjoyed the freedom they received to go in-depth in their way, to plan, and to create in their conditions, at their time, with independence and creativity that they can display, and yet they were calm from the fact that I was in the background to help at any time. The idea that took shape through their collaboration and dedication, their self-orientation, and the display of self-efficacy strengthened them even further for the continuation of their work.

"During our sessions I realized that all the students come, they all come on time, bringing what is necessary to bring and not forgetting, behaving in an adult manner and respecting one another, seeking the advice of one another, speaking appropriately and respectfully, and it is really possible to say that we have drawn close and learned to know one another more."

A student describes her surprise when she saw that the students came, with precision and responsibility, as if they were coming to a lecture, to lessons where they received freedom to work as they saw fit.

"The work in the group was enjoyable and fun, all the students contributed to the work, did what they were assigned. I was surprised that truly they all cared and they all came to the meetings as if this is a lesson with a lecturer and this is something that is obligatory but since they all truly cared and it was important to all of them they came and we worked with exceptional cooperation. After the first lesson ended and we decided upon a topic, which is the ratio, each one was asked to take articles and read them well and to learn them in the best possible way, with the goal that we would become knowledgeable in the material."

The freedom given incentivized them, and at any given moment they knew that they could turn to me for advice and mediation.

"Throughout all the work, we were given a high degree of freedom, which helped us work under our conditions and develop our creativity with a free hand. We found the time that was comfortable for us to work, for the most part on Sundays, and we were not limited by guidelines. Beyond this, we knew that we always had somebody to turn to as necessary and sometimes we received the help of the lecturer and we were able to receive constructive criticism and the opinion on the continuation of the work."

They enjoyed seeing how an idea matures into a product.

"As we advanced in the sessions, I saw how a small idea becomes a reality, and how the thought we had in our mind is created in front of our eyes. We understood that to create a product on the highest level, we must learn the material and understand it, including all the small nuances. We also investigated the topic of shapes but more than this we investigated which elements must be in the board game so that it will be educational, will convey the material, and will facilitate the learning. We took the relevant articles, we summarized them, and we implemented them in the final product".

In parallel to the autonomy, there was my close accompaniment as a lecturer; mediation and feedback that served as formative assessment.

"We were given full autonomy, including in the choice of the topic and the subtopics and the choice of the platform through which we wanted to present the project. During the project, despite the autonomy, we presented the project in the intermediate stages to the lecturer in order to obtain feedback and partial guidance so that we can be precise in the creation of the product, primarily in terms of the mathematical field. We were given the autonomy on the part of the lecturer to decide what is good for us. There were a number of times that we asked whether we are advancing in the right direction. And we understood that yes, we are."

The counseling, the mediation, and the focus they received from me, along with their responsibility and dedication, brought them to wonderful results, and this strengthened them to teach their pupils in this way.

"This course contributed to me knowledge and tools for work in groups. I felt that although in the PBL project there is the giving of autonomy to every student, there was also direction as well as focus on the part of the lecturer. Through the entire process, we were very independent. Every Thursday we sat in the library for the continuation of the product without Miri supervising us. We felt very dedicated to the product and we wanted to create the best product that we could. For this, we needed to cooperate and to persevere in the work and this we did. After I attempted practical and involving learning and I will be happy to adopt this strategy in the future as well."

Many students reported that the autonomy that they received strengthened in them the leadership ability, the responsibility, the initiative, and the creativity. Their self-confidence in their efficacy and self-orientation rose tremendously, and in the future they will implement this way of teaching because of their belief in it and will teach their students by giving them the utmost autonomy.

"Following this course I learned how the approach in which the pupils (and the students) are given the freedom to search, to think, to investigate, to organize, and to

initiate personal projects becomes the learning for active inquiry in which they experience, research, search for what is suitable through the implementation of multiple considerations. In this approach, the motivation increases and the curiosity strengthens, and this causes the better flow and enjoyment during the learning."

This student also tells with pride that she discovered she has self-orientation and that the course developed in her traits of leadership.

"When we ended the entire story and I read it as a whole and I saw all the apparatuses that belong to it, I felt satisfaction with my work and I understood that I have high self-efficacy and personal orientation for every project in which I participate. In addition, the course strengthened in me the principle of leadership and entrepreneurship and developed in me my thoughts about different creative ideas that I can use as a teacher in the future."

This student also tells that in the work on the project she discovered she had abilities she had not known existed.

"In this course I discovered by myself many things that I had not known about myself previously, I discovered considerable creativity in myself, I did not think and I did not know beforehand that I have so many ideas and that I can participate in the writing of a book."

Along with the autonomy that developed in them creativity and responsibility and leadership, the close guidance I provided was important to them.

"The course developed initiatives and projects, empowered in me things like responsibility, caring, leadership, and most importantly for me creativity. The lecturer Miri was with us in every stage. We worked in some of the lessons alone, but she was updated in all that we are doing and she was available for every question and to help, she supported and encouraged us throughout the entire process."

The students who enjoyed the autonomy reported that they do not remember from the past such freedom in learning. They felt that I am relying on them and on the ideas

that they will choose, and nevertheless I am a listening ear and I provide counsel as necessary.

"We were given absolute autonomy by the lecturer. At first, we did not really understood how this should look and here the lecturer came in and explained what needs to be and not how. The topic we chose, how the topic will be presented we chose. Miri only served for us as a listening ear. The feeling was that there is no lecturer who knows everything and we attempt to 'satisfy' her and to do what she wants us to do but that the work is ours and we do what we think is right. I do not remember any time during the studies that I received so much autonomy. This feels like somebody depends on you an on your ideas."

In a certain group, because of the autonomy the members received, they decided on rules of their own for work on the project, since three of the group members came from another group where things were not good for them (this was an isolated and unusual case). In this group, there was pleasant harmony throughout the entire process.

"We chose to define clear rules for the work on the project and they are: we listen to one another, we share ideas and information, we talk quietly and respectfully, we let each one of the members of the group express her opinion, we let each one in the group take an active part in performing the task, we give into one another, we compromise as necessary, we are helped in abilities of each one so as to perform the task effectively, and last we strive together to achieve the goal. In the entire process, we understood that we learn to take responsibility for our actions, we learn to maintain a schedule. And most important, in our opinion, we learn that a group always achieves more than does the individual."

Student Satisfaction

From my observations, most students reported tremendous personal satisfaction as well as satisfaction with the opportunities to think outside of the box, with the long and in-depth process they experienced, with the autonomy they were given, with the solving of the problems they encountered on their way, with their transformation into knowledge

experts in the mathematical skills they chose, with the collaboration, with the friendship created in the group, and with the mathematical product they built and presented to their classmates.

Many students told that the feelings of success and personal satisfaction were not felt during the presentation of the final product but accompanied them throughout the entire process. Some students were astounded to discover the creativity that was innate in them, like the students in the Orchard Group who were excited by themselves when they planned a teaching aid from a tree that illustrates different geometric shapes that are opened in every stage. When the product was prepared after it was built by the carpenter uncle of one of the students, they decided to register it as a patent.

Others were enthusiastic in every opportunity in which they found solutions to the difficulties that arose along the way, like the members of the Kingdom of Polygons Group, who felt in one of the lessons considerable frustration but rather quickly recovered, were encouraged, and found solutions. In this lesson, as in many others, groups presented in front of everyone their progress in the project. After every group report, a short feedback was performed of the students and me, but this time it was rather critical on the part of the students on the inaccuracy in the properties of the polygons as they wrote in the story. The feedback was justified, since the writing in rhymes and humor came at the expense of the mathematical accuracy. The way in which things were said by two students deeply hurt the members in the group and caused them to feel frustration and despair. After the feedback, in the time allotted beforehand to the short mediation meetings with me, I sat with them and showed them that it is not necessary to write the story again. They saw that with great ease it is possible to find accurate alternative sentences and to correct minimally the story so that it will continue to be fascinating, filled with rhymes, and primarily accurate in geometric terms. Their enthusiasm after the corrections that they found themselves and their personal satisfaction were tremendous.

It was apparent that the satisfaction was felt as the students evinced greater expertise in the chosen mathematical contents. The learning and depth in the

mathematical skills through the reading of articles on the topic and the engagement in activities that the textbooks offer caused broad understanding of the difficulties in it and the thinking on the appropriate strategies, so that they will provide a solution to all the students, including those with difficulties. In the Red Planet Group and in the Quadrilateral Group it was interesting to see the way in which they processed the mathematical skill. In the initial group sessions each one of the group members brought an article she found on the topic and read to all of them the sentences that were emphasized in marker. The student who was in the role of the teacher focused the main points and in the end of every session she summarized to them the properties of the quadrilaterals and the important aspects that should be taken into account in the work on the project.

In addition, the collaboration and responsibility of each one of the members of the group contributed to the social and professional experience. In some of the groups, such as the Orchard Group and the Monopoly of Three Dimensional Objects Group, the girls did not know one another. However, the cooperation in the work on the project made them into good friends. They indicated that in the future they will be a work group at any opportunity in the college and will remain friends forever.

All the groups described the product as formative moments, moments of success and of achievements, moments accompanied by a sense of contentment, pride in the product, satisfaction with the creative and professional presentation they prepared, and self-confidence in the mathematical skills they studied in-depth. In the Orchard group they decided to register the geometric product they created as a patent. Other groups, such as the Kingdom of Polygons Group, the Multiples of 7 and 8 Group, and the Red Planet Group thought to publish mathematical books they wrote in rhymes or through origami.

Because of these formative moments and in light of the enthusiasm of their friends, the students who observe and are active understood that this is the method of teaching they will use in the future with their students in mathematics and in other areas of knowledge. Some even received reinforcement for their success even before the

summative event, when they presented their products to their own children or to their students, and the enthusiasm on the part of everybody was tremendous. One of the students said, "If I enjoyed the process, then there is no reason why the students will not enjoy."

They saw that the creativity is expressed not only in the ability of drawing or art ability but also, for instance, in the leadership ability (who was in the role of the teacher in the group), technological ability (who built the illustration films in the special applications on the iPad), the ability of planning, and so on. In addition, there were students who following the course began to believe in their self-efficacy, in their personal orientation for learning, and in their ability to cope well with situations of uncertainty.

The following quote shows the student's satisfaction that he received instruments that not every teacher in the field receives. With this new knowledge he can teach the pupils in a new and meaningful way that is not routine in the elementary schools.

"I feel that I am going with an instrument of teaching that not everyone has outside. This is a significant way of teaching of great importance. A child who will be taught in this way will have a different learning experience that requires responsibility, depth, and collaboration. These are things that are not trivial for children of the elementary school.

The students are proud of the final product that was created but no less of the process they experienced.

"It should be noted that aside from the final product that turned out on a very high level the work process until the arrival at the product itself is very important. This is a way that includes many ups and downs, disappointments and successes. This is a work method that caused the group unification and responsibility of the members of the group for one another."

In this quote the student appears satisfied with the innovative learning he acquired. He believes that he will teach his pupils in this way and thus they can develop, challenge themselves, and develop their imagination.

"Since the course was different from other courses and allowed me to learn in a way that is not routine it awakened in me considerable enjoyment. After the end of the project, I am satisfied with the product and with the cooperation that there was among the group members. The course gave me an opportunity to experience something else and to know another way of teaching (through projects) and evaluation of the pupils. Learning through projects certainly will awaken among the pupils curiosity and motivation in learning. In addition, it will challenge the pupils and will develop their imagination. To conclude, I am happy to teach pupils using projects and I will let them as much as possible to try work of this type (if possible)."

Here the autonomous cooperative learning is presented as enjoyable and effective.

"In general, the work was challenging but enjoyable, primarily we enjoyed because of the fact that we were 'masters of ourselves' and we chose alone what to do and how. In my eyes, independent inquiry learning is the most effective learning. I think that the course caused me to open up to new things and even taught me new things about myself. I greatly enjoyed the learning and the experience I experienced during the course. I learned that there are new ways to teach and to convey the material that require considerable time and many resources but they are effective and significant. I want of course to thank the girls who were a part of the active learning. There is no doubt that without the cooperation of everyone we would not have reached the final product that in my opinion came out very nice and interesting. I greatly enjoyed the tremendous experience I had during the course and as a teacher in the future I would like to integrate this type of learning in my class."

The students indicate that they learned during the presentations from the other groups. They were thrilled by the various presentations of the products and the processes, by the diversity and the creativity.

"Since we were responsible for 'something that was not beforehand', I felt a sort of fraternity, since only we, the girls of the team, felt every little advance in the project. I can also add that I greatly enjoyed the presentation of our product but no less seeing the works of the other groups. It was fascinating to see how every group succeeded in

creating a product that was completely different from the product of another group and in getting inspiration for different and diverse ways of teaching."

The reasons for the personal satisfaction of the students are presented in the following table. The data are from my observations and from the reflections, interviews, and observations of the students.

Table Number 4: Reasons for Personal Satisfaction

Groups	Thinking outside of the box	In-depth process	Collaboration & friendship	Autonomy & self-responsibility	Becoming experts in the mathematical skill	Finding solutions for difficulties on the way	Pride in the final product
Ratio Clips	3	3	3	3	3	3	3
Quadrilateral Family	3	3	3	3	3	3	3
Orchard	3	3	3	3	3	3	3
Angle in the Clock	1	2	3	1	2	1	3
Multiples of 7 and 8	3	3	3	3	3	3	3
Monopoly of Three Dimensional Objects	1	2	3	1	2	1	3
Fibonacci in the Olive Grove	3	3	2	3	3	3	3
The Exploding Box	3	3	3	3	3	3	3
The clock Application	3	3	3	3	3	3	3
The Red Planet	3	3	3	3	3	3	3
Gil and the Circles	3	3	3	3	3	3	3
Revolving Three Dimensional Objects	3	3	3	2	3	2	3
Kingdom of Polygons	3	3	3	3	3	3	3
Teaching Kit for the Circle	3	3	3	2	3	2	3

* The table was created following their reflections, observations, and interviews and following my observations.

In addition, I asked the group members to rank the reasons for satisfaction from 1 to 3.

Legend:
1 – Satisfaction on a low level
2 – Satisfaction on a moderate level
3 – Satisfaction on a high level

It can be seen according to the table that nine groups of the fourteen groups felt satisfaction at a high level because of all the reasons mentioned: the opportunity to think outside of the box, the in-depth process they experienced, the collaboration and friendship in the group, the autonomy and responsibility they obtained over themselves, their transformation into experts in the mathematical skill, the finding of solutions to the problems, and their pride in the final product they created.

In the Fibonacci Group there were tensions and frictions at the beginning among the group members because of their assertiveness and desire to achieve control. From the moment I mediated among them, the atmosphere in the group calmed down and it was even possible to see a good partnership form among them. However, I would not define this as true friendship, as was created in other groups, such as the Orchard group and Monopoly of Three Dimensional Objects Group.

The Revolving Three Dimensional Objects Group and the Teaching Kit for the Circle Group displayed lack of confidence in themselves in light of the autonomy and self-responsibility they obtained. This lessened their enjoyment and their personal satisfaction. Even after they solved problems and difficulties along the way, the concern about the autonomy and responsibility troubled them and I can say even delayed them in the first lessons.

It can also be said that in the Monopoly of Three Dimensional Objects Group and the Angles of the Clock Group the fear of autonomy and responsibility they felt inhibited them and it can be said that this even completely stifled them in the first lessons and did not allow them to advance in the project. The girls in the two Druse or Christian Arab groups did not succeed in overcoming by themselves the initial difficulties, the lack of mastery of the mathematical skills, and the need for thinking outside of the box. Only in the continuation, with my mediation, when they became experts in the mathematical skills did they begin to enjoy the process. They were especially proud of their products and even more of the deep friendship created among them.

VI. Discussion of the Research Results

Theme 1

In this chapter, I will discuss the findings that arose in the previous chapter and that derive from the students' reflections, observations, and interviews and describe the socio-psychological environment in the course, the students' feelings, and their opinions regarding project based learning. These findings are supported in this chapter by the evidence from the review of the literature according to the two main themes.

'Freedom in the framework' (Harpaz, 2015). The active and involving learning environment through projects in mathematics as encouraging the development and cultivation of the skills of the 21st century: investigative and applied learning, relevant and authentic, critical thinking, and formative assessment, collaboration and belonging to the group, academic seriousness, and relationship with experts.

The heterogeneity of the classes is accepted and more understood because of the psychological research of the period. The Multiple Intelligences Theory of Gardner (1996) and the Emotional Intelligence of Goleman (1997) extended the need for diverse holistic and unique ways of learning that education is required to develop alongside the intellect and constructivism. The constructivist theories developed from the perception formed by Piaget and furthered by Vygotsky (2000) and others and maintain that people structure meaning actively and on the basis of previous knowledge. The constructivism has become the accepted learning perception and following it new perceptions of teaching and learning have developed (Perkins, 1995). Harpaz (2015) calls these perceptions the 'third model', which is mainly freedom in a framework.

Today the educators understand that they need to help the pupils leave the school with skills suitable for the 21st century, such as cooperation, creativity, teamwork, problem solving, and making decisions, so that they can learn, work, adjust, flourish, and succeed in an unknown future (Cervantes, Hemmer, & Kouzekanani, 2025). One of the ways to develop these skills is through Project-Based Learning – PBL.

Already from the beginning of the work process on the project, the students are required to investigate the specific mathematical skill they chose, so as to go in-depth in it and master it. The active inquiry continues throughout the entire process, when in the continuation they learn and discover from the information sources effective strategies and appropriate teaching aids for teaching the mathematical skill. This is an opportunity to go in-depth in the search for diverse information sources, ranging from articles and journals, through books and Internet websites, to consultation with the knowledge experts in the field.

After they have become experts on the topic through reading, practice, and mathematical discourse, they think about the applied product and begin to plan and build it. The process of learning through the building of projects is constructivist in nature, since it is enabled because of previous knowledge and experience and enables active and dialogue inquiry learning adjusted to everyday life

Pitt and Kirkwood (2010) wanted to improve the mathematics achievements of their students through the use of nontraditional ways such as constructivism, in order to answer the needs of time. In their argument, the curriculum in mathematics does not need to come only from the Ministry of Education or the district but from daily planning that takes into consideration the needs of the students, teachers, topic, and content. In their eyes, the goal of mathematical education is to create an environment that supports the problem solving and investigation. They qualify and say that it is necessary to remember that this pattern is not suited to every learning environment. The observations of teachers and tests showed a considerable improvement in the learners' achievements and proof of the importance of the constructivist teaching in mathematics as well as proof of the importance of teaching based on the student's previous knowledge and experience, encouragement of the discussion and guidance of the students to structure knowledge actively, and use of the students' ideas and their centrality in the class (Pitt & Kirkwood, 2010).

In the group, they think about the mathematical skill and the product that will serve as an applicative teaching aid for this skill. However, for the product to be relevant

and meaningful for the learner, it is important that the instructor give her approval before there is progress in the process.

Sometimes the product built is relevant but not authentic in its nature because of gaps in the mathematical knowledge or because of a non-charismatic leader in the group. To prevent lack of authenticity, the instructor should intervene in the building of the group and should make certain there is at least one dominant student. However, the instructor must make certain that the group members learned the mathematical skill and researched it in-depth before they began with the planning of the product..

Rahm (2011) maintains that for the educational system to be relevant to the 'third wave', a real cultural change in the classroom is necessary that will make it a community of learners. This change includes dialogic relationship – a productive dialogue between the teacher and the pupils, among the pupils themselves, and between the pupils and the knowledge (Hecht & Rahm, 2008) and relevance to the community and the environment – value-oriented learning related to topics, problems, dilemmas, and main themes.

The many drafts written by the students who lead in the group, the written reflections and the consultations with experts, the instructor's feedback in the meetings with the group members, and the feedback in the assembly constitute a central tool in the formative assessment and a best time framework for critical thinking. When these tools are deployed with sensitivity and effectiveness, they cause the motivation to continue to work on the product. Following the summative event of every group, it is possible to inspire again critical thinking and to perform summative assessment of the project and the product.

In essence, the evaluators and the providers of feedback are all the students in the feedback sessions in the assembly, the leading student, the student in the role of the observer, the instructor, and additional experts. The students noted that the observer in every group sees from the side the discussions and can evaluate the temporary situation in an objective manner, and therefore they proposed to exchange it in a rotation from week to week so that all the group members will experience the observation from the side, the critical thinking, and the evaluation that is not made possible with active participation in

the discussions in the group. In this way, the observers' awareness increases with the internal observation of themselves and the observation of the atmosphere in the group and the contribution of the group members to the atmosphere.

Patton (2012) maintained that when pupils know that their project will be presented to people outside of the classroom, then something changes in the way in which they work on it from the very first moment. They understand that they will need to stand beside their work - in both senses of the word - to provide explanations and to answer the questions of their peers, family members, and even strangers.

This fact inspires in them greater motivation and commitment than does the desire to 'get a good grade'. In addition, the event constitutes an opportunity for their family members, as well as for people from the wider circles of the community, to see from up close what happens in the school walls and to take part in the school experience, which leads to the reinforcement of the relation between the school and its community.

Multiple drafts have considerable value, even when the aim is to evaluate the personal work of each and every pupil, since they provide an excellent means for the teacher to understand the pupil's process during his work on the project. It is possible to see the starting point, his progress and improvement from the first draft, through the following drafts, until the final product. This perspective is important for all the pupils and especially for pupils with learning difficulties or language difficulties.

A culture of rewriting and improving multiple drafts creates a significant change in the way in which pupils address their work and even themselves. The next stage in this change is that the pupils will give feedback to one another on their work, instead of all the projects being submitted to the teacher, for her exclusive judgment.

In project-based learning, time is dedicated especially to this feedback. This is an opportunity for every pupil to learn from the work of his classmates and to receive feedback for his work in a structured and guided manner, which does not threaten and is not upsetting. In the providing of the feedback it is important to be sensitive, considerate, precise, relevant, and in parallel to make suggestions for improvement.

When the students are allowed to assemble into groups for work on their own project, they connect to their known friends. In this way, their self-confidence rises. Their success in the project is influenced primarily by the leader in the group and not by the friends or the cultural homogeneity in the group.

When the leader is dominant, there is in-depth and productive learning, and in the work on the product there is an effective division of roles according to abilities and skills. In addition, a charismatic leader determines cooperative work procedures already from the beginning and when people act according to them, group responsibility, flow, and the best progress are felt.

Marwan (2015) sought to examine the nature of teaching English with information communication technology (ICT) and PBL. He saw that the learning becomes interesting, structured, significant, and rewarding for the learners. The teachers create a social atmosphere among their pupils, easily follow up after their pupils, and encourage activities of learning with peers through questions and corrections, and the obtained results is more effective learning. They prove good understanding of pedagogy and bring interesting materials and aids to teaching, and the learners assume responsibility and self-direction for their learning, investigate, and complete the tasks enthusiastically. They are active and responsible, they make decisions, and they are motivated and confident in their learning. Because of the social atmosphere, it appears that the teachers and the learners are partners in the learning and in the making of decisions.

A research study conducted in a university in Malaysia in 2012 compared between two groups of pupils. One group studied in the conventional method, while the other group studied using project-based learning. A significant change was apparent in terms of the performances in the favor of the group that studied using project-based learning. This change was made possible because of the characteristics of project-based learning, such as cooperative learning, learning in small groups, discussions in groups, the presence of the teacher for help in learning, and resources for problem solving (Tarmizi & Bayat, 2012).

It is apparent that the students' general satisfaction and personal satisfaction are felt when they display greater expertise of the chosen mathematical contents. The learning and the increased depth in the mathematical skill through the reading of article on the topic and engagement in activities that the textbooks or Internet websites suggest cause the broader understanding of the difficulties inherent in it and thinking about the appropriate teaching strategies, so that they will provide an answer to all the students, including those with difficulties.

Every group needs the mediation of the instructor for aesthetic corrections, or in other words, for technical direction or for mathematical accuracy. The intensity of the mediation is different from group to group and occurs during the increased focus on the mathematical skill or during the group feedback or during the feedback in the assembly. The groups that especially needed the instructor are the groups that had not mastered the mathematical topic and did not have a charismatic leader in the group. It appears that the mediation of the instructor or any instructor in the required field of knowledge raises the level of curiosity, motivation, enthusiasm, active inquiry, and activity.

The educator Steinberg (in Patton, 2012) formulated a set of six principles for the planning of project-based learning: authenticity, academic rigor, applied learning, active exploration, adult relations, and assessment that includes many opportunities for reflection, both for teachers and for students, for holding at the end a summative event, such as an exhibition, show, or presentation, and for meeting standards accepted in the real world and in the school.

Theme 2

"Your approach, not your talent, will determine how far you reach." (Zigler) The empowerment of the student in active and involving learning through projects in mathematics – from fears, frustrations, and difficulties to curiosity, motivation, self-direction, and satisfaction.

In the beginning of the work on the project, most of the students who are not accustomed to such work report many concerns because of feelings of vagueness,

confusion, fear, stress, concern, curiosity, frustration, and even skepticism about the project' success. Most of them have many questions about the continuation, some consider the project itself difficult and complicated, and meeting the time deadlines with the group partners (also from another culture – Jewish and Druse) for the production of the best product only further complicating. Their memories from the work in the group and from the success in this work are not especially positive. The most serious concerns are created because of the open and vague guidance (in their words, "walking into the unknown" and "navigating in the dark" or because of the lack of mastery of the mathematical skills. Concerns about the lack of technical abilities, the multiplicity of members in the group, the lack of discipline, and multiple differences of opinion appear to influence to a lower extent the getting stuck in the work.

A qualitative research conducted in South Africa examined during a year the project-based learning approach. The findings indicated that the exposure of the students to teaching in this way advanced in them significant patterns of learning that are characterized by the processing of topic critically and by processes that aim at independent learning and self-regulation over time. They used planning, problem analysis, product testing, adjustment and reflection of solutions, and metacognition. It seems that the students were less dependent on the lecturer. However, it was clear that there was tension on the matter of the change to the new approach. Some students insisted on obtaining correct and precise answers in the lecture and not through their experience (Malan, Ndlovu, & Engelbrecht, 2014).

Many students feel frustration and difficulty when they are given open and vague instruction for work on a project, since they are not accustomed to self-direction in their learning, to active inquiry, and to collaboration. Another frustration comes when the gaps in the knowledge and the lack of mastery of the mathematical skill are apparent. Therefore, they cannot see in their minds the final product in the first stage or they discover that a certain idea cannot develop into a meaningful product. Sometimes these frustrations cause arguments in the group; however, from session to session things become clear and the frustration vanishes.

Additional frustrations and difficulties in more advanced stages are technical and content-related. When a group receives criticism about its product in one of the feedback sessions in the group or in the assembly, the confidence is again upset, but the instructor's sensitivity and direction for corrections can restore the group's confidence and motivation to continue. For technical difficulties such as bugs in the application or in the mathematical language, lack of time for group meetings, and the decision for work materials effective solutions are found with the mediation of experts in the topic.

In large groups there are uncontrollable laughs, or small talk unrelated to the work, differences of opinion, and arguments. The students themselves agree that there needs to be a maximum of four or five members in the group and they recommend that the role of the observer will move in rotation so that everybody will feel the atmosphere from the side and will become more responsible.

Teachers in a research study of Braran and Maskan (2009) received an opportunity to practice the method and found it to be beneficial. Their conclusion was that the programs for the training of teachers will give pupils opportunities to experience the method and practice it.

The teachers argued that PBL will develop abilities for the performance of the research, work in a group, and productivity (Larmer & Mergendoller, 2010). PBL was found to help pupils develops skills appropriate for the 21st century, such as research, search for information, cooperation, communication, critical thinking, and use of technology. However, the difficulties that the teachers indicated in the implementation of the program were the pupils' lack of interest, limited time, and problems in class management. The pupils could not analyze the workload in the project, wasted time, and left everything to the last minute. In this manner the pupils could not acquire the required skills.

The teachers maintained that they find it difficult to implement the method when they are not experienced in the planning of the stages but they believe that they will learn to do so and that the differences in the learning styles of the different pupils needs to be

taken into account. This invites thoughts about the quality and effectiveness of the training of teachers in the field of PBL (Baysura, Altun, & Yucel-Toy, 2016).

Students who exhibit curiosity and are not afraid of the open and vague guidance and of the autonomy given to them believe in their efficacy for self-direction. They are not stuck but advance with great curiosity in the learning of the mathematical skills, in the organization, and in the planning of the product until its formation.

The motivation of the students derives to a similar degree from eight main reasons: the existence of a charismatic leader in the group, membership and collaboration in the group, an exciting idea that arises or materials available for the mathematical skill, the need for experience in active learning on projects to teach their students in the future, the lecturer's mediation, the expertise in the mathematical skill, positive feedback, and pride in the formed creation (see the findings: the Orchard Group that is attempting to take out a patent on the product it created).

In the groups in which considerable motivation for work was apparent, as well as the self-direction of the participants, there is a clear and charismatic leader who sweeps everybody to work. His leadership implements the group members, unites them, and creates flow.

Autio (2009) proposes that it is necessary to create internal motivation already in the early stages through an encounter with a cognitive conflict, even if this appears more difficult than the creation of motivation on the basis of outside factors. However, Ryan and Deci (2000) assume that external motivation can change into internal if the project is sufficiently interesting. Moreover, brainstorming, non-routine activities, high level thinking, and creative problem solving will be practiced already in the lower classes. When the pupils begin in the planning stage with beneficial and relatively simple applications, their motivation is increased.

The group members for whom the granted autonomy was not strange enjoy the freedom to go in-depth in their way, to plan, and to create under their conditions, in their

time, with independence and creativity. Nevertheless, they still are calm about the fact that the instructor is in the background to help them at any time.

Many students also reported that the autonomy they receive empower in them the ability of leadership, responsibility, self-direction, initiative, creativity, and self-confidence. Most of the students also reported tremendous personal satisfaction and high satisfaction with the opportunity to think outside of the box, with the long and in-depth process they experienced, with the autonomy given to them, with the solving of problems they encountered on their way, with their transformation into knowledge experts in mathematical skill they chose, with the collaboration, with the friendships created in the group, and with the mathematical product they build and presented to their friends in the class. Many students told that the feeling of success, personal satisfaction, and general satisfaction were not only when they presented the final product but accompanied them throughout the entire process. Some students were thrilled to discover the creativity that was hidden in them in various areas: painting, technological abilities, leadership abilities, problem solving, and more.

Hakkarainen and Vapalahti (2011) examined learning through video in contexts of project based learning and saw that in this learning there was something illustrative and authentic. This supported most of the traits of meaningful learning for problem solving. The self-reporting of the pupils about their emotional involvement was positive: enthusiasm, happiness, interest, and sense of community were the most common and intensified emotions. This was an encouraging outcome from their perspective since positive emotions predict high academic achievements.

Twelve groups from the fourteen groups received a score of 5 in all the parameters in the table of the evaluation of the mathematical products: mathematical accuracy, authenticity and creativity, effectiveness and usefulness, aesthetics, and quality of the product (according to the four levels of thinking of the Supervision of Mathematics). In contrast, the other two groups, Angles of the Clock and Monopoly of Three Dimensional Objects, received a score of 5 only in the level of accuracy of the mathematical product.

It appears that these two groups which did not receive the highest score in all the parameters in the evaluation of the mathematical product did not feel satisfaction as did the members of the other groups with the autonomy they were given, the opportunities to think outside of the box, or the solutions they found on the way. They did not display curiosity, self-direction, and willingness to research, to read articles and learning materials, and to consult experts. The cooperation in both of these groups was expressed primarily in the small talk and less in the learning and critical thinking about their progress. They felt anxiety and frustration with the "unknown" at the start of the work on the project. The members of the Monopoly of the Three Dimensional Objects group also were afraid about their lack of knowledge on the topic of three dimensional objects.

VII. Conclusions, Implications, and Recommendations

The active and involved learning through projects in mathematics integrates into it active inquiry of mathematical skills and strategies towards the preparation of authentic and meaningful products such as games and teaching aids suited to this skill and their implementation in the elementary schools. The students become learners who are investigative and self-directed, who cooperate with their fellow members of the group, who empower and encourage one another, who develop and cultivate by themselves the skills of the 21st century and learn to believe in themselves and in their abilities.

In the work on the project, there must be guidance to build an innovative and authentic product that is relevant to its user. The instructor should be confident in their expertise in mathematical skill and then he must make certain of the originality and relevance of the idea and confirm them before starting work on the mathematical product. In addition, it is important that the instructor integrate at least one leading student in every group and make certain that the group members learned the mathematical skill and researched it in-depth before they began with the planning of the product.

When expertise in the mathematical skill is felt, the group participants are filled with confidence, are satisfied, and are motivated to continue the work. However, all the groups need the mediation of the instruction at different levels and at different stages of the work on the project, both in the increased depth in the mathematical skill and in the group feedback or feedback in the assembly for aesthetic corrections, or in other words, for technical direction or mathematical precision. Collaboration and the best teamwork do not necessarily depend on friendship and homogeneity but are made possible because of the composition of a group with different abilities, when at least one student is charismatic and possessed of leadership abilities, and when procedures are determined ahead of time. When the leader is dominant, there is in-depth and productive learning, and in the work on the product there is an effective division of roles according to abilities and skills. In addition, a charismatic leader determines cooperative work procedures already from the beginning and when people act according to them, group responsibility, flow, and the best progress are felt, even if there is a social, cultural, or cognitive

heterogeneity. When for some reason there is no student who has charisma in the group and who functions as the "group leader" or when there are significant gaps in the knowledge, the instructor must help more in all the required parameters, in mediation, in management, in the inculcation of the knowledge, in the raising of ideas for the product, and in their implementation.

It is important to appoint in every group an observer who will document the atmosphere in the group. However, it seems that it is important that the observer be changed in an organized rotation from meeting to meeting, or in other words, in every meeting another group member will function as the observer. In this way, additional critical thinking is implemented among all the group participants and the sense of personal responsibility is strengthened, thus improving the joint work. In this way, frustrations are avoided, and the students exhibit high level of motivation to work on the project, curiosity, interest, depth in the contents and their structuring, inquiry, and strong connection to their work.

The many drafts written by the students who lead in the group, the observers' reports, the written reflections and the consultations with experts, the instructor's feedback in the meetings with the group members, and the feedback in the assembly constitute a central tool in the formative assessment and a beneficial time framework for critical thinking, and following them there are aesthetic or content changes for the improvement of the product. When these tools are deployed with sensitivity and effectiveness, they cause the motivation to continue to work on the product. In the summative events before a diverse audience, the students feel tremendous pride and following the feedback on their presentations there is further critical thinking, summative assessment, collaboration, and dissemination of knowledge and values.

Fears, anxieties, and frustration among students derive primarily because of the ambiguity in the instructions for work on the project (from their statements: "navigating in the dark" and "walking into the unknown") or because of the lack of mastery of the mathematical skills. These sometimes may cause lack of curiosity and motivation, and lack of progress and may even cause a certain getting stuck in the work. Additional

concerns are the difficulty with language, the lack of technological skills, the autonomy given to them, the concern about lack of creativity, the multiplicity of members in the group, the lack of discipline, multiple differences of opinion, and lack of familiarity with the group members, which appear to influence to a lower extent the getting stuck in the work. In any event, this feeling of getting stuck is released only through additional clarifications of the instructor in the group encounter or through the continuation of her close guidance.

Students who are not afraid of the open and vague guidance discover great curiosity regarding the process, the mathematical skills, and the expected product. This curiosity is what motivates them to choose the mathematical skill, to learn it in-depth, and to organize and to plan the product until its formation.

The work obligates the students to cooperate already from the initial stages, to go in-depth in the reading about the mathematical skill, to practice it, and to develop the skills of the independent learner. The motivation is at its peak when there is the optimal cooperation, or when expertise in the mathematical skill is apparent, or when there is a prominent leader who functions as the "teacher" of the group and motivates it, or when they decide on the idea for the final product, are excited about it, and are proud of it, or from the lecturer's mediation or from the autonomy granted, or when they receive positive feedback, or for some of the reasons I listed together.

Personal feelings of satisfaction are felt during all the stages in the process and not only in the feeling of pride in the presentation at the end. The students are enthusiastic when they discover their ability to work in situations of uncertainty, their innate creativity (in art, in technical ability, in leadership, or in problem solving), their thinking outside of the box, their expertise in mathematical skills, their ability to solve problems with the autonomy given to them, with increased responsibility, and with collaboration.

It appears that the members of the group who were afraid and frustrated at the start of the work with the feeling of "going into the unknown" and the gaps in the mathematical skill in the continuation do not feel satisfaction with the autonomy they

were given, the opportunity to think outside of the box, or the solutions that appear along the way. In addition, they do not exhibit curiosity, self-direction, and willingness to research, to read articles and learning materials, and to consult experts. The cooperation in both of these groups was expressed primarily in the small talk and less in the learning and critical thinking about their progress. At the end of the process, in the evaluation of the mathematical product it appears that they do not excel in all the parameters.

Of the fourteen groups, only two groups were identified as such. When the instructor identifies such a group, he must bridge and direct already at the start of the process towards the increased depth in the mathematical skill, clarification, organization, raising of ideas, and choice of a charismatic leader for the group and provide extensive feedback as well as encouragement and reinforcement.

Because of these formative moments and in light of the enthusiasm of their fellow students, who observe and are active in feedback of in the summative events, the students understood that they attempted and experienced work on a mathematical project, since this is the way of teaching they will use in the future with their pupils in mathematics and in other areas of knowledge. Some even received reinforcement of their success before the summative event, when they presented their products to their own children or to their pupils, and the enthusiasm of all was tremendous. One of the students said, “If I enjoy the process, there is no reason why my pupils will not enjoy.”

In light of the research conclusions and despite the difficulties mentioned, I recommend the direction of suitable resources that include significant budgets and in-depth training for teaching through projects in mathematics. Thus, in the school there will be learning in mathematics that is more active and involved, through projects and problems.

Furthermore, it would be interesting to examine the parameters that I examined in this research study also on projects in diverse and integrated knowledge areas.

References

ALLEN, P.A. (2009) *Membership Roles in Field Research*, Thousand Oaks, CA: Sage.

ALSHARIF, K. (2014) How Do Teachers Interpret the Term 'Constructivism' as a Teaching Approach in the Riyadh Primary Schools Context? *Procedia – Social and Behavioral Sciences*, 141. 1009-1018.

ANFARA, V. A., BROWN, K. M., & MANGIONE, T. L. (2002). Qualitative Analysis on Stage: Making the Research Process More Public. *Educational Researcher, 31*, 7, 28-38.

ASIKAINEN, M. A., PEHKONEN, E., & HIRVONEN, P. E. (2013) Finnish Mentor Mathematics Teachers' View of the Teacher Knowledge Required for Teaching Mathematics, *Higher Education Studies*, 3(1).

AUSLANDER, S. S., SMITH, S. Z., SMITH, M. E., HART, L. C., & CAROTHERS, J. (2016) A Case Study of Two Groups of Elementary Prospective Teachers' Experiences in Distinct Mathematics Content Courses, *SRATE Journal*, Summer, 25 (2).

AUTIO, O. (2009). Pedagogical Background for Technology Education – Meaningful Learning in Theory and Practice, *i-manager's Journal of Educational Technology*, 5l (4l), January – March, 14-23.

BARAN, M., & MASKAN, A. K. (2009) Proje tabanlı öğrenme modelinin fizik öğretmenliği ikinci sınıf öğrencilerinin elektrostatiğe yönelik tutumlarına etkisi [The Effect of Project Based Learning Approach on the Second Year's Pre-service Physics Teachers' Attitudes Toward Electrostatics]. *Dicle Üniversitesi Ziya Gökalp Eğitim Fakültesi Dergisi*, 12, 41-52.

BARBASH, M. (2006) What We Teach When We Teach Mathematics and How Is This Supposed to Be Reflected in Teacher Training? *Journal of the Mofet Institute*, 58, 54-61. (Hebrew)

BAROODY, A. J. &. COSLICK, R. T. (1998) *Fostering Children's Mathematical Power: An Investigative Approach to K-8 Mathematics Instruction*. Mahwah, NJ: Lawrence Erlbaum Associates.

BARROWS, H. S. (1992) *The Tutorial Process*. Springfield, IL: Southern Illinois University School of Medicine.

BASHAN, N. (2012) *Learning Connected to the Beat of Life, 'You and I Will Change the World' - Project-Based Learning*. The Democratic Education Institute. (Hebrew).

BASS, H. (2005) Mathematics, Mathematicians, and Mathematics Education, *Bull. Amer. Math. Soc*, 4(4), 417–430.

BAYSURA, O. D., ALTUN, S., & YUCEL-TOY, B. (2016) Perceptions of Teacher Candidates regarding Project-Based Learning, *ANI - Eurasian Journal of Educational Research*, 62, 15-36.

BOROVIK, A. V. (2012) *Shadows of Truth: Metamathematics of Elementary Mathematics.* Working draft 0.822. November. Retrieved from: http://www.maths.manchester.ac.uk/~avb/ST.pdf

BROWN, C. A., & BORKO, H. (1992) Becoming a Mathematics Teacher. In D. A. Grouws (Ed.), *Handbook of Research in Mathematics Teaching and Learning* (pp. 209-242), New York: Macmillan.

BURNS, M. (2007) Nine Ways to Catch Kids Up. How Do We Help Floundering Students Who Lack Basic Math Concepts? *Educational Leadership*. 65 (3). November, 16-21.

CAMPBELL, P. F. (1997) Connecting Instructional Practice to Student Thinking. *Teaching Children Mathematics*. 4(2). October, 106-110.

CARPENTER, T. P. & LEHRER, R. (1999). Teaching and Learning Mathematics with Understanding. In Elizabeth, F., & Thomas, A. R. (Eds.) *Mathematics Classrooms that Promote Understanding* (pp. 19-32).

CARPENTER, T. P., FENNEMA, E. & FRANKE, M. L. (1992) Cognitively Guided Instruction: Building the Primary Mathematics Curriculum on Children's Informal Mathematical Knowledge. Paper presented at the American Educational Research Association Conference, San Francisco.

CARR, S. (2000) As Distance Education Comes of Age, the Challenge Is Keeping the Students. *The Chronicle of Higher Education*, February 11, A39-41.

CARR, W., & KEMMIS, S. (1984) [1983] Becoming Critical: Knowing through Action Research, Victoria: Deakin University Press.

CARUTH, G. (2014) Learning How to Learn: A Six Point Model for Increasing Student Engagement, *USA Participatory Educational Research (PER)*, 1(2), December.

CERVANTES, B., HEMMER, L., & KOUZEKANANI, K. (2015) The Impact of Project-Based Learning on Minority Student Achievement: Implications for School Redesign, *NCPEA Education Leadership Review of Doctoral Research,* 2 (2), October.

CHAMBERS, D. L. & HANKES, J. E. (1994) Using Knowledge of Children's Thinking to Change Teaching. In Aichele, D. B. (ed.). *Professional Development for Teaching Mathematics*, 1994 Yearbook of the National Council of Teachers of Mathematics. Reston, Va.: The Council, pp. 286-295.

CHESSNER, S. (2005) *The Child in the Armor: To Connect to the Child Suffering from ADHD and Behavioral Difficulties*. Ach Press. (Hebrew)

COCHRAN SMITH, M. (2003) The Unforgiving Complexity of Teaching: Avoiding Simplicity in the Age of Accountability, *Journal of Teacher Education*, 54(1), 3-5.
COCHRAN-SMITH, M. (2004) The Problem of Teacher Education, *Journal of Teacher Education*, 55(4), 295-299.

COCHRAN-SMITH, M., & VILLEGAS, A.M. (2016) Research on Teacher Preparation: Charting the Landscape of a Sprawling Field, in: Gitomer, H.D., & Bell, A. C., (Eds.), *Handbook of Research on Teaching* (5th ed.), 439-547.

COCHRAN-SMITH, M., CANNADY, M., MCEACHERN, K.P., PIAZZA, P., POWER, C., & RYAN, A. (2010) Teachers' Education, Teaching Practice and Retention: A Cross-Genre Review of Recent Research, *Journal of Education*, 191(2), 19-31.

COHEN, L., & MANION, L. (1989) *Research Methods in Education*, Second Edition, London: Routledge.

COHEN, M. (2013) *Policy of the Promotion of Meaningful Learning in the Educational System: Document for Constant Construction*, Ministry of Education, December 24. (Hebrew)

COPUR-GENCTURK, Y. (2015) The Effects of Changes in Mathematical Knowledge on Teaching: A Longitudinal Study of Teachers' Knowledge and Instruction**,** *Journal for Research in Mathematics Education*, May, 46(3), 280-330.

CORNELIUS, S. (2014). Facilitating in a Demanding Environment: Experiences of Teaching in Virtual Classrooms Using Web Conferencing, *British Journal of Educational Technology*, 45(2), 260-271.

CRESWELL, J. W. (1998). *Qualitative Inquiry and Research Design: Choosing among Five Traditions*. London: Sage Publications.

CSIKSZENTMIHALYI, M., & RATHUNDE, K. (1993) The Measurement of Flow in Everyday Life. *Nebraska Symposium on Motivation*, 40, 57–97

DATNOW, A., & PARK, V. (2009) Conceptualizing Policy Implementation: Large-Scale Reform in an Era of Complexity, in: Sykes, G., B. Schneider & D. Plank (Eds.), *Handbook of Education Policy Research*, New York: Routledge, 348-361.

DENZIN, N. K., & LINCOLN, Y. S. (2000) Introduction: The Discipline and Practice of Qualitative Research, In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of Qualitative Research* (Second Edition, pp. 1-28). London: Sage Publications.

DEWEY, J. (1969) Democracy and Education (Translated by: Y. T. Helman), Jerusalem: Bialik Institute. (Hebrew)

ELDAKAK, S. (2012). Introduction and Attributes of Meaningful Learning Using Technology (MLT), February 27.

EMBI, M. A. & ABDULLAH, A. (2009). Meaning Learning in a Networked Learning Environment. In M. A. Embi (Ed.) *Computer-Mediated Communication: Pedagogical Implications of Malaysian Research Findings* (pp. 128-141). Shah Alam: Karisma Publications.

ERNEST, P. (1993). The Impact of Beliefs on the Teaching of Mathematics. In P. Ernest (Ed.), *Mathematics Teaching. The State of Art* (pp. 249-254). New York: The Falmer Press.

FARES, M. (2016) Teaching Mathematics – Why? *Journal of the Mofet Institute*, 57, 3-5. (Hebrew)

FENNEMA, E., CARPENTER, T. P. & PETERSON, P. L. (1989) Learning Mathematics with Understanding. In Brophy, J. E. (ed.). *Advances in Research on Teaching*.

FLORES, M. A. (2006) Being a Novice Teacher in Two Different Settings: Struggles, Continuities, and Discontinuities. *Teachers College Record, 108*(10), 2021-2052. http://dx.doi.org/10.1111/j.1467-9620.2006.00773.x

FRIERE, P. (1981) *Pedagogy of the Repressed*, Jerusalem: Mifras. (Hebrew)

GARRETT ET AL. (2016) A Framework for Authenticity in the Mathematics and Statistics Classroom, *The Mathematics Educator*, 25 (1).

GAZIT, A. (2013) Attitudes of Student Teachers in Mathematics in the Elementary School towards the Use of Humor in Teaching, *Education and Its Environment*, 35, 115-125. (Hebrew)

GIVTON, D. (2001). Grounded Theory: Meaning of the Process of the Analysis of Data and Construction of Theory in Qualitative Research, in N. Sabar-Ben Yehoshua (Ed.), *Traditions and Currents in Qualitative Research* (pp. 195-228). Lod: Dvir. (Hebrew)

GREENE, M. L., & CAMPBELL, C. (1993) *Becoming a Teacher: The Contribution of Teacher Education*. Alberta Department of Education, Edmonton.

GREENFELD, N. (2015) Academia – Class and Teacher Training, *Journal of the Mofet Institute*, 55, 4-3. (Hebrew)

GUBA, E. G., & LINCOLN, Y. S. (1998). Competing Paradigms in Qualitative Research. In N. K. Denzin & Y.S. Lincoln (Eds.) *The Landscape of Qualitative Research* (pp. 195-220). London: Sage Publications.

GUBA, E. G., & LINCOLN, Y. S. (1998). *Fourth Generation Evaluation*. Newbury Park, CA: Sage Publications.

GUR-ZEEV, A. (1997) The Fundamental Problem of Critical Education in Our Time, *Studies in Education*, 2(2), 9-52. (Hebrew)

GUR-ZEEV, A. (Ed.) (1996) *Education in the Era of Postmodern Discourse*, Jerusalem: Magnes. (Hebrew)

HAIMOWITZ, M. L. (1989) *Human Development.* New York: Thomas Y. Crowwell. Kenya.

HAKKARAINEN, P., & VAPALAHTI, K. (2011) Meaningful Learning through Video-Supported Forum-Theater, *International Journal of Teaching and Learning in Higher Education*, 23(3), 314-328.

HANKES, J. E. (1996) An Alternative to Basic-Skills Remediation. *Teaching Children Mathematics*. 2 (8). April.

HARARI, H. (1992) *Report of the Supreme Committee for Scientific Technological Education*, Ministry of Education, Culture, and Sport. (Hebrew)

HARGREAVES, R. (1996) *Changing Teachers*, New York: Teachers College Press.

HARPAZ, Y. (2005). *Rod, Bait, and Fish: Approaches to the Education of Thinking*, Jerusalem: Branco Weiss Institute. (Hebrew)

HARPAZ, Y. (2008) *The Third Model: Teaching and Learning in the Thinking Community*, Tel Aviv: Sifriat Poalim. (Hebrew)

HARPAZ, Y. (2014) Howard Gardner, *Echo of Education*, 88(6), 50-56. (Hebrew)

HEBERT, E., & WORTHY, T. (2001). Does the First Year of Teaching Have To Be a Bad One? A Case Study of Success. *Teaching and Teacher Education, 17*, 897–911. http://dx.doi.org/10.1016/S0742-051X(01)00039-7

HECHT, Y., & RAHM, E. (2008) The Dialogue in Democratic Education: From Personal Empowerment to Social Activism, In A. Nimrod (Ed.) *Empowering Dialogues in Humanistic Education*, HaKibbutz HaMeuchad Press. (Hebrew)

HERSHKOWITZ, S., & BITON, Y. (2014) Computer Based Interactive Tasks - Innovative Approach to Assessment in Mathematical Education, In *Innovations and Measurements – On Measurement, Assessment, Technology, and the Relationships between Them*. (Hebrew)

HULING-AUSTIN, L. (1992) Research on Learning to Teach: Implications for Teacher Induction and Mentoring Programs. *Journal of Teacher Education, 43*(3), 173-180. http://dx.doi.org/10.1177/0022487192043003003

INTER-AMERICAN DEVELOPMENT BANK (IDB). (2011) Less Talk, More Play: Bolstering Math Learning in Argentina. *IDB Briefly Noted*. 9. June.

JEWPANICH, S., & PIRIYASURAWONG, P. (2015) Methods via Social Media Model for Enhancing Problem Solving Skills, *International Education Studies, 8 (6).*

JONES, B. F., RASMUSSEN, C. M. & MOFFITT, M. C. (1997) *Real-Life Problem Solving: A Collaborative Approach to Interdisciplinary Learning*. Washington, DC: American Psychological Association.

JOSSELSON, R. (2004). The Hermeneutics of Faith and the Hermeneutics of Suspicion. *Narrative Inquiry, 14*(1), 1-28.

KEAN, A. C., & KWE, N. M. (2014) Meaningful Learning in the Teaching of Culture: The Project Based Learning (PBL) Approach, *Journal of Education and Teaching Studies*, 2, 2, 189-197.

KEAR, K. ET AL. (2012). Web Conferencing for Synchronous Online Tutorials: Perspectives of Tutors Using a New Medium, *Computers and Education*, 1(1), 953-963.

KILPATRICK, W. H. (1918) The Project Method, *Teachers College Record*, 19, 319-335.

KINNEY, S. (2006) Action Research as a Paradigm – as an Outlook, In D. Levi (Ed.) *Action Research: Theory and Practice – Philosophical and Methodological Connections between Action Research and the Qualitative Research Paradigm,* Raanana: Mofet Institute. (Hebrew)

KNOHLL, M. (1995) The Project Method - Its Origin and International Influence, In: H. Rohrs & V. Lenhart (Eds.) *Progressive Education across the Continents,* Frankfurt: Peter Lang.

KOLB, D. A. (1984) *Experiential Learning: Experience as the Source of Learning and Development,* Englewood Cliffs: NJ: Prentice Hall.

KUH, G. D. (2006). Underrepresented Students Benefit Most from Engagement, Retrieved from: https://tle.wisc.edu/forum/underrepresented-studentsbenefit-most-engagement

KURUKKAN, A. (2015) *Learner and Teacher Perception on Difficulties in Learning and Teaching Mathematics: Some Implications*.

LACHNER, A., & NUCKLES, M. (2016) Tell Me Why! Content Knowledge Predicts Process-Orientation of Math Researchers' and Math Teachers' Explanations, ***Instructional Science,*** June, 44 (3), 221-242.

LARMER, J., & MERGENDOLLER, J.R. (2010) Seven Essentials for Project-Based Learning. *Educational Leadership, 68*(1), 34-37.

LESTER, F.K. JR., MASINGILA, J.O., MAU, S.T., LAMBDIN, D.V., DOS SANTON, V.M., & RAYMOND, A.M. (1994) Learning How to Teach via Problem Solving. In D. Aichele & A. Coxford (Eds), *Professional Development for Teachers of Mathematics*, (pp. 152-166). Reston, Virginia.

LINDQUIST, M. M. (1989) It's Time to Change. In P. R. Trafton and A. P. Shulte (Eds.), *New Directions for Elementary School Mathematics, 1989 Yearbook of the National Council of Teachers of Mathematics*, pp. 1-13, Reston, VA: NCTM.

LOUGHRAN, J. (2002) Effective Reflective Teaching: In Search of Meaning in Learning about Teaching, *Journal of Teacher Education*, 53(1), 33-43

MACK, M. & WESTENSKOW, J. (2014) The Evolution and Success of a Colorado High School through Project-Based Learning. *Principal Leadership*, October, 15 (2). 16-21.

MAHONEY, C. (2009) Teaching Culture in the Japanese Language Classroom: A NSW Case Study. *New Voices*, 3, 104-125.

MALAN, S. B., NDLOVU, M., & ENGELBRECHT, P. (2014) Introducing Problem-Based Learning (PBL) into a Foundation Programme to Develop Self-Directed Learning Skills. *South African Journal of Education*. 34(1).

MARION, S., & LEATHER, P. (2015) Assessment and Accountability to Support Meaningful Learning, *Educational Policy Analysis Archives*, 23(9).

MARKOVITZ, TZ. (2004) We Prepared Them, But Are They Prepared – Teachers Who Teach Mathematics Look at the Way They Were Prepared for This Job, Conference of Researcher Teachers, Oranim College, May 12. (Hebrew)

MARSHALL, C., & ROSSMAN, G. B. (1989) *Designing Qualitative Research*, London: Sage Publication.

MARWAN, A. (2015) Empowering English through Project-Based Learning with ICT, TOJET: The Turkish Online Journal of Educational Technology, 14 (42015), October.

MASLOW, A. H. (1954) *Motivation and Personality*. New York: Harper and Row.

MAYERS, P. (1978) *Flow in Adolescence and Its Relation to School Experience*. Ph.D. Dissertation, University of Chicago.

MCBER, H. (2000) *Research into Teacher Effectiveness.* Report for the Department of Education and Employment, UK.

MCMAHON, M. (1999) Are You Having Fun Yet?: Humor in the English Class. *The English Journal*, 88(4), 70-72.

MCNERNEY, C. (1994) A Model Preservice Program for Preparation of Mathematics Specialists in the Elementary School. In D. B. Aichele and A. F. Coxford (Eds.).*Professional Development for Teachers of Mathematics, 1994 Yearbook of the National Council of Teachers of Mathematics*, pp. 144-151.

MENDELSON, N. (2013) How Do Students Training for the Teaching of Mathematics in the Elementary School Perceive Their Professional Image? In D. Petkin & A. Gazit (Eds.) *The Mathematics Teacher: Training Characteristics, Knowledge, Teaching, and Personality of Teachers of Mathematics in the Elementary School* (pp. 293-321). Tel Aviv: Mofet Institute. (Hebrew)

MENGEL, T. (2010) *Undergraduate Education*, University of New Brunswick, 119-123.

MERRIAM, S. B. (1998) *Qualitative Research and Case Study: Applications in Education*. San Francisco: Jossey-Bass Publishers.

MEYER, K. (2014) Making Meaning in Mathematics Problem-Solving Using the Reciprocal Teaching Approach. *Australian Journal of Language & Literacy*, June, 37 (2), 7.

MILES, B. M., & HUBERMAN, A. M. (1984) Drawing Valid Meaning from Qualitative Data: Toward a Shared Craft. *Educational Researcher*, 13 (5), 20-30.

NACHIMUTHU, K., & VIJAYAKUMAR, G. (2011) Role of Educational Games Improves Meaningful Learning, *i-Manager's Journal of Educational Technology*, 8 (2), July-September.

NASLUND-HADLEY, E. (2011) Less Talk, More Play: Bolstering Math Learning in Argentina. *IDB Briefly Noted*. 9.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS (1989) *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: Author.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS (1991) *Professional Standards for School Mathematics*. Reston, VA: Author.

OSMANOGLU, A., ISIKSAL, M., & KOC, Y. (2015) Getting Ready for the Profession: Prospective Teachers' Noticing Related to Teacher Actions. *Australian Journal of Teacher Education, 40*(2). http://dx.doi.org/10.14221/ajte.2015v40n2.3

PAPICK, I. J., BEEM, J. K., REYS, B. J., & REYS, R. (1999) Impact of the Missouri Middle Mathematics Project on the Preparation of Prospective Middle School Teachers. *Journal of Mathematics Teacher Education, Vol. 2*, 301-310.

PATTON, A. (2012) *Work that Matters - The Teacher's Guide to PBL - Project-Based Learning*. (Hebrew)

PEHKONEN, E. (1993) What Are Finnish Teacher Educators' Conceptions about the Teaching of Problem Solving in Mathematics? *European Journal for Teacher Education, 16*(3), 237-256.

PEKER, M. (2016) Mathematics Teaching Anxiety and Self-Efficacy Beliefs toward Mathematics Teaching: A Path Analysis. *Educational Research and Reviews*, 11(3), 97-104.

PERKINS, D. (1997) *Towards the Intelligent School: From Practice of the Memory to Education of the Thinking, Jerusalem: Ministry of Education and the Branco Weiss Institute. (Hebrew)*

PESHKIN, A. (2000) The Nature of Interpretation in Qualitative Research. *Educational Researcher, 29*(9), 5-9.

PINKER, S. (2002) *The Blank Slate*. New York: Penguin Books.

PINTO, J. (2013). What Project-Based Learning Looks like in Math, November 13, Retrieved from: http://www.teachthought.com/learning/project-based-learning/project-based-learning-in-math-6-examples/.

PITT, J. & KIRKWOOD, K. (2010) How Can I Improve Junior Level Mathematics Achievements Using Constructivism? *Ontario Action Researchers*, 10 (3).

RAHM, E. (2011) *Project Based Learning Community: The Pedagogy for the Third Wave*. (Hebrew)

RAHM, E. (2011) Project-Based Learning: Pedagogy for the Third Age. In Rozenberg, E. (ed.). *Intermediate Call* 18, *Malam,* Association and the Ministry of Education. (Hebrew)

ROBLYER, M. D. (1999) Is Choice Important in Distance Learning: A Study of Student Motives for Taking Internet Based Courses at the High School and Community College Levels. *Journal of Research on Computing in Education*, 32(1), 157-171.

RYAN, R.M., & DECI, E.L. (2000). Self-determination Theory and the Facilitation of Intrinsic Motivation, Social Development and Wellbeing, *American Psychologist,* 55, 68-78.

SABAR-BEN YEHOSHUA, N. (Ed.) (2001) *Traditions and Currents in Qualitative Research*, Ganei Aviv – Lod: Dvir Press. (Hebrew)

SABAR-BEN YEHOSHUA, N. (Ed.) (2016) *Traditions and Currents in Qualitative Research: Perceptions, Strategies, and Advanced Instruments*, Raanana: Mofet Institute. (Hebrew)

SALUVEER, E. (2004). *Teaching Culture in English Classes*, M.A. Thesis. Retrieved on March 20, 2012 from http://www.lara25.com/mywebdisk/CI-EP/Saluveer.pdf.

SAVEY, J. R. & DUFFY, T. M. (1985) Problem Based Learning: An Instructional Model and Its Constructivist Framework. *Educational Technology*, 35 (5), 31-38.

SCHMIDT, W. H., BLOEMEKE, S., & TATTO, M. T. (2011) *Teacher Education Matters: A Study of Middle School Mathematics Teacher Preparation in Six Countries*, New York: Teachers College Press.

SEIDMAN, I. E. (1991). *Interviewing as Qualitative Research: A Guide for Researchers in Education and the Social Sciences*. New York: Teachers College Press.

SELLERMEIER, M. (2016) Action Research in Education: History, Characteristics, Criticism, In: N. SABAR-BEN YEHOSHUA, N. (Ed.) *Traditions and Currents in Qualitative Research: Perceptions, Strategies, and Advanced Instruments*, Raanana: Mofet Institute, p. 307. (Hebrew)

SEZER, R. (2008) Integration of Critical Thinking Skills into Elementary School Teacher Education Courses in Mathematics, *Education*, March.

SHARF, R. (2014) Data Based Teaching, *Free Lesson*, 109, 24, June. (Hebrew)

SHARF, R. (2014) Learning through Doing – A Journey around the World. *Free Lesson; Learning Environments*. 108 (3). (Hebrew)

SHARF, R. (2015) Playing with Numbers, *Free Lesson*, 112, 25, January. (Hebrew)

SHEFFIELD, L. J. (2003) *Extending the Challenge in Mathematics: Developing Mathematical Promise in K-8 Students*. Thousand Oaks, CA: Corwin Press.

SHEM TOV, N. (2015) *Improvising Teaching – It's Not a Rude Word: Theater Improvisation as a Tool of Teaching and a Way of Knowing*, Tel Aviv: Mofet Institute. (Hebrew)

SHKEDI, A. (2003) *Words that Try to Touch: Qualitative Research – Theory and Application*, Raanana: Ramot. (Hebrew)

SHḴEDI, A. (2005) *Multiple Case Narrative: A Qualitative Approach to Studying Multiple Populations*. Amsterdam and Philadelphia, PA: John Benjamins.

SHKEDI, A. (2011) *The Meaning behind the Words – Methodology in the Qualitative Research: Theory and Practice*, Ramot Press: Tel Aviv University. (Hebrew)

SHULMAN, L. S. (1986) Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher, Vol. 15*, 4-14.

SMITH, G. G., & FERGUSON, D. (2004) Learning Math Problem-Solving in Online Courses. *Proceedings of E-Learn 2004: World Conference on E-Learning in Corporate, Healthcare & Higher Education*, Washington, DC.

SMITH, G. G., & FERGUSON, D. (2005) Student Attrition in Mathematics E-Learning. *Australasian Journal of Educational Technology*, 21(3), 323-334. http://www.ascilite.org.au/ajet/ajet21/smith.html.

SMITH, G.G., FERGUSON, D., & CARIS, M. (2002) Teaching Online Versus Face-To-Face. *Journal of Educational Technology Systems*, 30(4), 337-364.

SMITH, M. S. & STEIN, M. K. (1998) Selecting and Creating Mathematical Tasks: From Research to Practice. *Mathematics Teaching in the Middle School*. 3(5). February. 344-350.

STAKE, R. E. (1995) *The Art of Case Study Research*, London: Sage Publications.

STAKE, R. E. (2000) Case Studies, In N. K. Denzin & S. L. Lincoln (Eds.) *Handbook of Qualitative Research* (Second Edition) (pp. 435-454) London: Sage Publications.

STEFAN, M. A., & POPESCU, A. M. (2014) Forming Future-Teacher Students Using Constructivist Theory in E-Learning Effectively, *eLearning and Software for Education*, 3, 365-369.

STEPIEN, W. J., GALLAGHER, S. A., & WORKMAN, D. (1993) Problem-Based Learning for Traditional and Interdisciplinary Classrooms. *Journal for the Education of the Gifted*, 16, 338-357.

STEPIEN, W., & GALLAGHER, S. (1993) Problem-Based Learning: As Authentic As It Gets. *Educational Leadership*, 51, 25-28.

STOCKERO, S.L. (2008) Using a Video-Based Curriculum to Develop a Reflective Stance in Prospective Mathematics Teachers, *Journal of Mathematics Teacher Education*, 11, 373-394.

STORM, K. J. (2015) Teaching as Assemblage: Negotiating Learning and Practice in the First Year of Teaching, *Journal of Teacher Education*, 66(4), 321-333.

STRAUSS, A. L. (1987). *Qualitative Analysis for Social Scientists*. Cambridge, UK: Cambridge University Press.

SUPERVISION OF MATHEMATICS, MINISTRY OF EDUCATION (2015). *Objectives and Work Plans*. Ministry of Education, Israel. (Hebrew)

SWELLER, J. (1994) Cognitive Load Theory, Learning Difficulty, and Instructional Design. *Learning and Instruction*, *4*(4), 295-312.

TARAN, J. (2011) *The Story of the Weave*. The Hebrew Reali School in Haifa. Efi Meltzer Ltd. (Hebrew)

TARMIZI, R. A. & BAYAT, S. (2012) Collaborative Problem-Based Learning in Mathematics: A Cognitive Load Perspective. *Procedia – Social and Behavioral Sciences*. 32. 344-350.

TASHAKKORI, A., & TEDDLIE, C. (1998) *Mixed Methodology: Combining Qualitative and Quantitative Approaches*, Thousand Oaks: Sage Publications.

THOMAS, J. W. (2000) A Review of Research on Project-Based Learning. http://www.bie.org/index.php/site/RE/pbl_research/29.

THOMAS, J. W., & MERGENDOLLER, J. R. (2000) Managing Project-Based Learning: Principles from the Field. Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans.

THOMPSON, E. (1997) Distance Education Drop-Out: What Can We Do? In R. Pospisil & L. Willcoxson (Eds.), *Learning through Teaching*: *Proceedings of the 6th Annual Teaching Learning Forum* (pp. 324-332), Murdoch University, February. Perth: Murdoch

TIMSS REPORT (2007) *Chapter 1: Mathematics Frameworks: Cognitive Skills Required for Mathematics.*

TOFFLER, A., & TOFFLER, H. (1984) *The Third Wave,* Tel Aviv: Am Oved. (Hebrew)

TOFFLER, A., & TOFFLER, H. (2008) *Revolutionary Wealth,* Tel Aviv: Am Oved. (Hebrew)

U.S. DEPARTMENT OF EDUCATION, OFFICE OF THE ASSISTANT SECRETARY FOR EDUCATIONAL RESEARCH AND IMPROVEMENT. (1984) *School Mathematics: Options for the Future. Chairman's Report of the Conference*. Washington, D.C.: U.S. Government Printing Office.

UHLMANN, E., & SWANSON, J. (2004) Exposure to Violent Video Games Increases Automatic Aggressiveness, *Journal of Adolescence,* 27, 41-52.

VACC, N. N., & BRIGHT, G. W. (1994) Changing Preservice Teacher-Education Programs. In D. B. Aichele and A. F. Coxford (Eds.). *Professional Development for Teachers of Mathematics, 1994 Yearbook of the National Council of Teachers of Mathematics,* pp. 116-127.

VEENMAN, S. (1984) Perceived Problems of Beginning Teachers. *Review of Educational Research, 54*(2), 143-178. http://dx.doi.org/10.3102/00346543054002143

WANG, J., & FUJINO, S. (2015) Cache-Cache Comparison for Supporting Meaningful Learning, 12th International Conference of Cognition and Exploratory Learning in a Digital Age (CELDA 2015), 332-336.

WARMAN, H. (2014) The Socratic Method and the Importance of Teaching Conflicts in the Subject of Citizenship – Illustration of a Dialectic Presentation of Civil Issues, *Statements,* 7, 181-202.

WASLEY, P. (2006) Underrepresented Students Benefit Most from Engagement, *The Chronicle of Higher Education, 53, A39.*

WATSON, D., & CLARK, L. A. (1988) *Development and Validation of Brief Measures of Positive and Negative Affect: The PANAS Scales*.

WEINER, S. (2014) What Can Be Expected from Somebody Who Teaches Mathematics in the Elementary School? In: D. Petkin & A. Gazit (Eds.) *The Teacher of Mathematics: Training Characteristics, Knowledge, Teaching, and Personality of Teachers of Mathematics in the Elementary School,* Ranana: Mofet Institute. (Hebrew).

WIDEEN, M., MAYER-SMITH, J., & MOON, B. (1998) A Critical Analysis of the Research on Learning to Teach: Making the Case for an Ecological Perspective on Inquiry, *Review of Educational Research*, 68(2), 130-178.

WILKINS, M. M., WILKINS, J. L. M., & OLIVER, T. (2006) Differentiating the Curriculum for Elementary Gifted Mathematics Students. *Teaching Children Mathematics*, 13 (1), August, 6-13.

WILLIAMS, S. M. (1992) Putting Case-Based Instruction into Context: Examples from Legal and Medical Education. *The Journal of the Learning Sciences*, 2, 367-427.

WOODS, P. (1996) *Researching the Art of Teaching: Ethnography for Educational Use*, London: Routledge.

XIAOQIN, L. (2016) Motivation Management of Project-Based Learning for Business English Adult Learners, *International Journal of Higher Education*, 5 (3).

Appendices

Appendix 1: Example of a Semi-Structured Interview

Hila David

Hila David is 27 years old. She has a combined degree in anthropology and sociology.

I always worked with children and with technology in smart classes. In the private lessons, I sought creative ways to teach them English and mathematics. I did not know project-based learning, but I always looked for ways to teach children through games, presentations, and activities, and not only through the book, the notebook, or copying from the blackboard. Last year, I was a substitute teacher. Once a week I took them to the computer classroom for applications.

There I encountered my greatest trauma – geometry. I thought that it is hard for me since I had a teacher who taught monotonously. I think about the experience in which I will teach my students and my experience. Where I am the most afraid, there will be the best things. I had concerns at the start of the course. An unfamiliar place to walk on unstable ground, an unfamiliar area, I don't know how deep it is needful to go. The language needs to remain very professional. You go to a place that you believe in but you are not sure of. Every time anew, this is investigation, this is trial and error.

I believe in the way but I did not understand how I was to implement it. I read the guide you sent, I understood what you are talking about, but still what topic I am to investigate in-depth, to be perfect in the smallest terminology … how do I do this? This is to advance, then to go back again, and again move another few steps, and again to go back a few steps. A type of spiral. There is here considerable creative thinking, and not only mathematical, something that will attract, when they will go back to this with the addition of material, they will remember that they saw it, we saw that this will remain, will make an impression, the ideas of the Princess Parallelogram. Let's assume that the rectangle is straight straight straight straight … I believe that if the learning is interesting

and experiential, it leaves an impression. Depth in the topic is an advantage, we used the books that we learn from every day, we touched upon articles that we would not address anyways. All the time we thought another step ahead, what more it is possible to know, this let us deepen existing knowledge. To add knowledge that there isn't. This is a tool that we know now, and we can use it in the continuation, a tool that goes into the basket of tools that we have and others do not have, and it is not possible to take it from us. A tremendous meaningful advantage and we will disseminate in the continuation.

Other teachers who did not experience this cannot alone. If they were to take an in-service training course in this … it would change the system.

We got to know one another during it. Shira unfortunately left the studies. Hagit sent a message that she is shifting to another group, where she feels slightly more dominant. This disappointed me – the way in which she behaved… when we remained four, every time we built more and more … I wrote another bit, every one took … there was wonderful interaction. There was enthusiasm, we wrote on the blackboard and Sophie copied … the freedom gave us independent and responsibility. There was some surprise in that not everybody saw. We took the lesson time and the recess time and then there were three and a half hours.

Freedom will not cause the children to scatter. This is their responsibility. This is mine. I am proud of it. I will present it to the teacher and the parents. This is their choice. There are no games. At the end, as a pupil I will know that I was not contemptuous, but rather I was responsible and I can be proud.

Difficulties: The feedback took the wind out of our sails. It was hard for us, this was our baby. This was difficult. Somebody criticizes something of yours that you had invested one thousand percent in. We said this is constructive criticism, even if it sometimes was sharp. Let's take a deep breath, let's hear what they say, we understood that there is logic in this, the criticism is sharp and incisive is important. Not everything is wonderful … this is the hard part. The group got along wonderfully, there is an amazing and lasting relationship, there was no room for frustration, since there was understanding, sometimes understanding after silence. This rhyme is more suitable. We found an

available class and we took over the blackboard. When you write on the blackboard, you write here and there, not like on the computer.

The inquiry – We looked for who would be the characters, searching on the Internet and in the guidebooks. We wrote on the traits of all the characters, we looked at the article of Van Hila, we saw how it was possible to do more things. If it didn't work out, we returned to the book, to see that the sentences are suitable.

Satisfaction is really high. After yesterday that you approved us and we are supposed to present in the next week. It is a bit hard to write a reflection.

Evaluation junctures – cross checking between books, and we greatly relied on your approvals three times. We became more critical and added hinting words, only, also … the high level of precision was added to the basket. We have drafts from the teacher and observer reports. We all did a little of everything. We helped the observer write. I said, she did, you want that I will write instead of you, I will enjoy working like this like a teacher. What I remember is mosaic work with geometric shapes from small stones, a memory from my elementary school.

Appendix 2: Example of the Drafts of the Student Teacher from the Ratio Project

The Drafts of the Student 'Teacher'

First Meeting: December 1, 2016

We came to the meeting with articles on the topic of the ratio towards the sixth grade. The articles include the definition of the concept of ratio, explanation on the ratio – different theories of the same situation, how to help children understand ratios, ratio and proportion, articles teaching how to teach ratio in the sixth grade (intuitive thinking of children in solving problems of ratio, and program for teaching ratio).

We read all the articles, we explained to one another, we received much knowledge on the topic and guidance for learning in the class. We emphasized all the

important sentences in the articles we read. We raised questions and dilemmas from the field and how to cope with them. We spoke about the mathematical concepts related to the topic of the ratio. On this topic, too, there were a number of dilemmas, such as whether the topic of greater than and less than suits the topic of the ratio. In addition, we thought how we will plan the product. There were conflicts about it. Some wanted an electronic presentation and others a book.

Second Meeting: December 8, 2016

We planned the process that would be performed in the field: we defined the goals that we want to implement in the product, steps, and measures of success.

A. Project Topic: Ratio for the Sixth Grade

B. Project Description:

Operative Learning Goals on the Topic of the Ratio

1. The student will know the concept of the ratio.
2. The student will know the form and reading and writing of the ratio.
3. The student will compare between quantities that are parts of the whole.
4. The student will compare between quantities that are a part and a whole.
5. The student will write a ratio suitable for the description/illustration of a datum.
6. The student will know to find a missing quantity according to a given ratio.
7. The student will know another meaning for a ratio fraction.
8. The student will find a division of a quantity according to a given ratio.

Concepts for the Topic of the Ratio

1. Reduction and expansion of the ratio
2. Multiplication and division.
3. Greater than, less than.
4. Greater by, less by.
5. Ratio
6. Simple fractions

Acquired Skills

1. Basic mathematics skills – reduction and expansion, multiplication and division, ratio, and so on.
2. Skills that contribute to functioning in everyday life.
3. Cognitive skills and logical thinking skills – drawing conclusions, representation of knowledge, positing hypotheses, examination and proof of hypotheses, generalization and analysis

Essential Questions

How will we perform the product? How will we perform the division of roles? On what skills and concepts will we focus? Will we perform material of all the subtopics of the ratio or only one subtopic extensively?

Time Schedule

Every week we dedicated one day in which we worked continuously for two hours.

Setting the Assessment Processes

- In what way will the presentation be performed? We speak each one in short on the process and on the products.

Criteria for the Success of the Project

- The activity needs to be challenging and to attract the students' attention.
- The presentation needs to be short and to the point.
- The presentation needs to be colorful and from the children's world.
- The films need to interest the students who will perform learning through projects.

Third Meeting: December 15, 2016

We decided to build a product built from films that we are preparing on our own. One of the girls will be the teacher, one will film, and the rest of the girls will be the pupils.

The films will be in a presentation built of a number of slides. The topic of the ratio begins from the inculcation of the concept of the ratio, what is the ratio, continues to the ratio between the quantities, then the finding of the quantity according to the given ratio, and so on.

These films will guide the teachers who will want to use the product in their classes. They will implement the film and will ask the pupils questions relevant to the topic. Every film will have a continuation film that is built from the response of the filmed pupils.

The goal of these films is to inspire the pupils' curiosity and to illustrate to them in the best possible way what is the ratio and to help the teacher in the teaching unit on the topic of the ratio.

Appendix 3: Example of the Protocol of Meetings of the Student Observing in the Ratio Project

Protocol from Meeting 4 – December 22, 2016

A conversation in which the lecturer is involved. Elinor the teacher and the lecturer talk and Elinor asks questions so as to understand the lecturer's desire. The lecturer gives her an example and uses descriptions when speaking with body movements and changes tones in the story, apparently so as to inspire curiosity in the students and attention. The girls symbolize the understanding by nodding their heads.

Gal intervenes in the conversation to express her opinion. Elinor responds to her so that she will understand. Karin speaks in parallel. A confused atmosphere is created but in the end there is understanding.

Elinor says there is a certain level of difficulty through rhyming activity, and Gal explains that the opposite is true. This is not hard and the lecturer reinforces her and then activity on this principle comes into the conversation but with data and an easier level of question.

The group holds a spectacular dialogue to form a decision pertaining to the activity that they chose to undertake. One raises an idea, and the other has a complementary idea, thus creating harmony and agreement in the group.

The lecturer gives the group a feeling of security. She calms the group, helps, and gives advice on activities. It appears that she is in the matter, using the textbooks, looking, and trying to find something else.

There are a number of girls who are focused on their small talk, and they do not appear to be so focused. Periodically they return to attention on the interesting project we are working on.

In the group there is no shame to ask questions and to express the opinion. Karin asks questions and seeks to know which examples will be given. Gal answers her. There are moments when the group incorporates humor that gives the motivation to continue and to act.

In the group there is a word problem on the ratio. The idea comes up about a story. The lecturer begins, the girls add their say. Sometimes the teacher negates it, and expresses her opinion in the negative after the students express an opinion. “No, why complicate this?”

Now there is a discussion of the films. It was summarized to have opening films that present the problem and result film. The group formed the idea, and at the moment works on the content, gives a basis, receives the lecturer’s approval. The lecturer helped and now it appears that the pressure in the group has decreased.

The teacher Elinor takes command and gives the division of the roles to the group so as to promote the project.