

SUPPLEMENTAL MATERIAL FOR: Rovibrational levels of HD

Krzysztof Pachucki^{1, *} and Jacek Komasa^{2, †}

¹*Institute of Theoretical Physics, University of Warsaw, Hoża 69, 00-681 Warsaw, Poland*

²*Faculty of Chemistry, A. Mickiewicz University, Grunwaldzka 6, 60-780 Poznań, Poland*

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The following tables collect components of the dissociation energy for all rovibrational levels of HD. The abbreviations used within the table:

BO – the Born-Oppenheimer component

AD – the adiabatic correction

NA – the nonadiabatic correction

α^2 – the relativistic correction of the order α^2

α^3 – the QED correction of the order α^3

α^4 – the QED correction of the order α^4

Total – the total dissociation energy, D_0

δ – the uncertainty assigned to D_0

*Electronic address: krp@fuw.edu.pl

†Electronic address: komasa@man.poznan.pl

TABLE I: Components of dissociation energy of HD

$v \setminus J$	0	1	2	3	4	5	6	7	8	9	10	11	12
0 BO	36401.9332	36312.6626	36134.7364	35869.3700	35518.3506	35083.9965	34569.1058	33976.8983	33310.9519	32575.1373	31773.5525	30910.4600	29990.2284
0 AD	4.2509	4.2870	4.3587	4.4653	4.6056	4.7779	4.9805	5.2111	5.4676	5.7473	6.0477	6.3661	6.6998
0 NA	-3267	-3345	-3500	-3731	-4037	-4414	-4862	-5376	-5955	-6594	-7290	-8040	-8840
0 α^2	-5300	-5319	-5358	-5490	-5751	-6158	-6627	-7160	-7759	-8414	-9125	-9894	-10714
0 α^3	-1964	-1956	-1941	-1918	-1888	-1852	-1809	-1760	-1705	-1646	-1582	-1515	-1444
0 α^4	-0016	-0016	-0016	-0016	-0016	-0015	-0015	-0015	-0014	-0014	-0013	-0013	-0012
0 Total	36405.7828	36316.5549	36138.7136	35873.4735	35522.6205	35088.4710	34573.8214	33981.8890	33316.2492	32580.7701	31779.5467	30916.8389	29997.0121
0 δ	-0010	-0010	-0010	-0010	-0010	-0010	-0010	-0010	-0010	-0010	-0010	-0011	-0011
1 BO	32768.2136	32682.8029	32512.5786	32258.7206	31922.9637	31507.5569	31015.2138	30449.0553	29812.5476	29109.4390	28343.6959	27519.4427	26640.9053
1 AD	5.1835	5.2164	5.2818	5.3789	5.5065	5.6632	5.8471	6.0562	6.2882	6.5407	6.8112	7.0969	7.3954
1 NA	-9554	-9625	-9764	-9973	-10249	-10590	-10994	-11458	-11980	-12556	-13185	-13862	-14585
1 α^2	-5509	-5527	-5563	-5616	-5685	-5769	-5867	-5977	-6098	-6227	-6364	-6505	-6651
1 α^3	-1778	-1770	-1756	-1734	-1706	-1671	-1630	-1584	-1533	-1477	-1416	-1353	-1286
1 α^4	-0015	-0015	-0015	-0014	-0014	-0014	-0014	-0013	-0013	-0012	-0012	-0011	-0011
1 Total	32773.6224	32688.2505	32518.1034	32264.3604	31928.7546	31513.5337	31021.4092	30455.4998	29819.2695	29116.4637	28351.0463	27527.1389	26648.9644
1 δ	-0012	-0012	-0012	-0012	-0012	-0012	-0012	-0013	-0013	-0013	-0013	-0014	-0014
2 BO	29312.1246	29230.4963	29067.8197	28825.2412	28504.4452	28107.6153	27637.3848	27096.7816	26489.1669	25818.1743	25087.6479	24301.5843	23464.0782
2 AD	5.9800	6.0096	6.0685	6.1560	6.2708	6.4114	6.5763	6.7633	6.9702	7.1947	7.4343	7.6865	7.9487
2 NA	1.5306	1.5369	1.5495	1.5682	1.5930	1.6237	1.6600	1.7018	1.7487	1.8005	1.8571	1.9180	1.9831
2 α^2	-5677	-5694	-5727	-5775	-5839	-5916	-6006	-6106	-6216	-6334	-6457	-6585	-6716
2 α^3	-1603	-1596	-1582	-1561	-1535	-1502	-1458	-1419	-1371	-1318	-1261	-1201	-1138
2 α^4	-0013	-0013	-0013	-0013	-0013	-0013	-0012	-0012	-0012	-0011	-0011	-0010	-0010
2 Total	29318.9058	29237.3125	29074.7056	28832.2304	28511.5704	28114.9074	27644.8730	27104.4929	26497.1260	25826.4033	25096.1664	24310.4092	23473.2237
2 δ	-0015	-0015	-0015	-0015	-0015	-0015	-0016	-0016	-0016	-0017	-0017	-0017	-0018
3 BO	26030.1861	25952.2825	25797.0393	25565.5708	25259.5140	24880.9906	24432.5589	23917.1590	23338.0547	22698.7733	22003.0463	21254.7535	20457.8712
3 AD	6.6368	6.6631	6.7154	6.7928	6.8944	7.0186	7.1637	7.3278	7.5088	7.7044	7.9121	8.1293	8.3537
3 NA	2.0550	2.0607	2.0719	2.0887	2.1109	2.1384	2.1709	2.2082	2.2502	2.2965	2.3471	2.4015	2.4597
3 α^2	-5804	-5819	-5849	-5891	-5951	-6021	-6102	-6193	-6292	-6397	-6508	-6621	-6737
3 α^3	-1438	-1431	-1418	-1399	-1374	-1343	-1306	-1265	-1219	-1169	-1115	-1058	-9999
3 α^4	-0012	-0012	-0012	-0012	-0012	-0011	-0011	-0011	-0010	-0010	-0010	-0009	-0009
3 Total	26038.1525	25960.2800	25805.0987	25573.7219	25267.7856	24889.4100	24441.1515	23925.9482	23347.0616	22708.0166	22012.5422	21264.5155	20467.9102
3 δ	-0019	-0019	-0019	-0019	-0019	-0019	-0019	-0020	-0020	-0020	-0020	-0021	-0021
4 BO	22919.9321	22845.7170	22697.8358	22477.3726	22185.9200	21825.5420	21398.7266	20908.3334	20357.5360	19749.7648	19088.6496	18377.9659	17621.5852
4 AD	7.1493	7.1722	7.2175	7.2846	7.3724	7.4795	7.6042	7.7446	7.8986	8.0640	8.2383	8.4191	8.6038
4 NA	2.5308	2.5358	2.5458	2.5607	2.5804	2.6047	2.6335	2.6666	2.7038	2.7448	2.7895	2.8376	2.8890
4 α^2	-5887	-5901	-5928	-5968	-6019	-6082	-6155	-6236	-6323	-6416	-6513	-6611	-6710
4 α^3	-1283	-1277	-1265	-1247	-1223	-1193	-1159	-1119	-1076	-1029	-978	-925	-869
4 α^4	-0011	-0011	-0011	-0011	-0010	-0010	-0010	-0010	-0009	-0009	-0009	-0008	-0008
4 Total	22928.8940	22854.7061	22706.8788	22486.4954	22195.1476	21834.8977	21408.2320	20918.0081	20367.3975	19759.8282	19098.9275	18388.4681	17632.3192
4 δ	-0022	-0022	-0022	-0022	-0022	-0022	-0023	-0023	-0023	-0023	-0024	-0024	-0025
5 BO	19979.9863	19909.4475	19768.9047	19559.4140	19282.5272	18940.2548	18535.0209	18069.6113	17547.1188	16970.8875	16344.4575	15671.5143	14955.8415
5 AD	7.5122	7.5314	7.5696	7.6259	7.6994	7.7886	7.8919	8.0075	8.1333	8.2670	8.4063	8.5488	8.6918
5 NA	2.9589	2.9633	2.9721	2.9851	3.0023	3.0235	3.0486	3.0774	3.1097	3.1453	3.1840	3.2256	3.2698
5 α^2	-5925	-5937	-5961	-5996	-6042	-6097	-6161	-6231	-6307	-6387	-6469	-6552	-6634
5 α^3	-1138	-1132	-1121	-1103	-1081	-1053	-1020	-983	-943	-898	-851	-800	-748
5 α^4	-0010	-0010	-0010	-0009	-0009	-0009	-0009	-0009	-0008	-0008	-0008	-0007	-0007
5 Total	19989.7501	19919.2343	19778.7372	19569.3140	19292.5156	18950.3510	18545.2424	18079.9739	17557.6361	16981.5705	16355.3151	15682.5527	14967.0643
5 δ	-0025	-0025	-0025	-0025	-0025	-0026	-0026	-0026	-0026	-0026	-0027	-0027	-0027

TABLE IV: Components of dissociation energy of HD

$v \setminus J$	13	14	15	16	17	18	19	20	21	22	23	24	25
0 BO	29017.2781	27996.0344	26930.8865	25826.1524	24686.0517	23514.6829	22316.0069	21093.8357	19851.8251	18593.4714	17322.1113	16040.9253	14752.9423
0 AD	7.0460	7.4020	7.7653	8.1332	8.5031	8.8727	9.2397	9.6016	9.9564	10.3017	10.6357	10.9560	11.2608
0 NA	.9688	1.0581	1.1516	1.2489	1.3499	1.4543	1.5620	1.6728	1.7864	1.9029	2.0219	2.1435	2.2676
0 α^2	-6.6707	-6.872	-7.035	-7.197	-7.356	-7.511	-7.660	-7.803	-7.938	-8.065	-8.184	-8.299	-8.390
0 α^3	-1.371	-1.296	-1.218	-1.140	-1.060	-0.981	-0.901	-0.821	-0.742	-0.664	-0.586	-0.510	-0.436
0 α^4	-0.012	-0.011	-0.010	-0.010	-0.009	-0.009	-0.008	-0.007	-0.007	-0.006	-0.006	-0.005	-0.004
0 Total	29024.4839	28003.6768	26938.9769	25834.6998	24695.0621	23524.1599	22325.9517	21104.2469	19862.6992	18604.8024	17333.8914	16053.1441	14765.5876
0 δ	.0011	.0012	.0012	.0013	.0013	.0014	.0015	.0016	.0016	.0017	.0018	.0019	.0020
1 BO	25712.3594	24738.0860	23722.3320	22669.2785	21583.0144	20467.5161	19326.6330	18164.0773	16983.4184	15788.0815	14581.3488	13366.3643	12146.1411
1 AD	7.7039	8.0199	8.3406	8.6635	8.9861	9.3061	9.6209	9.9282	10.2259	10.5114	10.7827	11.0375	11.2734
1 NA	1.5352	1.6159	1.7004	1.7886	1.8801	1.9748	2.0726	2.1733	2.2767	2.3827	2.4912	2.6021	2.7153
1 α^2	-6.798	-6.946	-7.093	-7.237	-7.378	-7.513	-7.643	-7.765	-7.879	-7.984	-8.080	-8.164	-8.238
1 α^3	-1.217	-1.146	-1.073	-0.999	-0.924	-0.849	-0.774	-0.699	-0.625	-0.551	-0.479	-0.408	-0.338
1 α^4	-0.010	-0.010	-0.009	-0.009	-0.008	-0.008	-0.007	-0.006	-0.006	-0.005	-0.005	-0.004	-0.004
1 Total	25720.7960	24746.9115	23731.5555	22678.9061	21593.0497	20477.9601	19337.4841	18175.3317	16995.0699	15800.1215	14593.7664	13379.1463	12159.2718
1 δ	.0015	.0016	.0016	.0016	.0017	.0017	.0018	.0019	.0020	.0021	.0022	.0022	.0026
2 BO	22579.2735	21651.3207	20684.3401	19682.3921	18649.4529	17589.3962	16505.9809	15402.8425	14283.4896	13151.3045	12009.5467	10861.3606	9709.7857
2 AD	8.2182	8.4925	8.7689	9.0449	9.3180	9.5857	9.8455	10.0950	10.3319	10.5535	10.7575	10.9414	11.1025
2 NA	2.0521	2.1248	2.2010	2.2804	2.3630	2.4485	2.5367	2.6276	2.7209	2.8165	2.9144	3.0143	3.1161
2 α^2	-6.847	-6.978	-7.108	-7.234	-7.355	-7.470	-7.579	-7.679	-7.771	-7.852	-7.923	-7.982	-8.027
2 α^3	-1.072	-1.005	-0.937	-0.867	-0.797	-0.726	-0.656	-0.586	-0.516	-0.447	-0.380	-0.313	-0.249
2 α^4	-0.009	-0.009	-0.008	-0.008	-0.007	-0.007	-0.006	-0.006	-0.005	-0.005	-0.004	-0.003	-0.003
2 Total	22588.7509	21661.1387	20694.5047	19692.9066	18660.3180	17600.6100	16517.5390	15414.7380	14295.7132	13163.8441	12022.3880	10874.4865	9723.1764
2 δ	.0018	.0019	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026	.0026
3 BO	19616.4252	18734.4512	17815.9599	16864.9097	15885.1848	14880.5795	13854.7875	12811.3962	11753.8861	10685.6336	9609.9193	8529.9393	7448.8217
3 AD	8.5825	8.8132	9.0432	9.2699	9.4907	9.7031	9.9045	10.1023	10.2640	10.4169	10.5481	10.6549	10.7340
3 NA	2.5214	2.5863	2.6544	2.7253	2.7990	2.8752	2.9539	3.0347	3.1175	3.2022	3.2884	3.3760	3.4646
3 α^2	-6.852	-6.966	-7.076	-7.183	-7.284	-7.378	-7.464	-7.541	-7.608	-7.663	-7.706	-7.749	-7.789
3 α^3	-0.937	-0.874	-0.810	-0.744	-0.678	-0.612	-0.546	-0.480	-0.416	-0.352	-0.289	-0.227	-0.167
3 α^4	-0.008	-0.008	-0.007	-0.007	-0.006	-0.006	-0.005	-0.005	-0.004	-0.004	-0.003	-0.003	-0.002
3 Total	19626.7494	18745.0660	17826.8681	16876.1114	15896.6776	14892.3583	13866.8443	12823.7206	11766.4648	10698.4508	9622.9561	8543.1737	7462.2285
3 δ	.0022	.0023	.0023	.0023	.0024	.0024	.0025	.0026	.0026	.0027	.0028	.0028	.0029
4 BO	16823.4316	15987.4434	15117.5418	14217.6057	13291.4527	12342.8259	11375.3865	10392.7117	9398.2975	8395.5672	7387.8852	6378.5769	5370.9565
4 AD	8.7899	8.9747	9.1557	9.3302	9.4955	9.6491	9.7881	9.9099	10.0115	10.1051	10.1423	10.1649	10.1540
4 NA	2.9433	3.0005	3.0603	3.1225	3.1869	3.2533	3.3215	3.3912	3.4622	3.5341	3.6065	3.6789	3.7507
4 α^2	-6.808	-6.904	-6.995	-7.081	-7.160	-7.231	-7.293	-7.344	-7.383	-7.408	-7.419	-7.413	-7.389
4 α^3	-0.811	-0.752	-0.691	-0.630	-0.568	-0.506	-0.445	-0.384	-0.324	-0.264	-0.206	-0.150	-0.095
4 α^4	-0.007	-0.007	-0.006	-0.006	-0.005	-0.005	-0.004	-0.004	-0.003	-0.003	-0.003	-0.002	-0.002
4 Total	16834.4021	15998.6523	15128.9885	14229.2866	13303.3617	12354.9540	11387.7220	10405.2397	9411.0003	8408.4239	7400.8713	6391.6642	5384.1125
4 δ	.0025	.0026	.0026	.0026	.0027	.0027	.0028	.0028	.0029	.0030	.0030	.0031	.0031
5 BO	14201.2798	13411.6920	12590.9340	11742.8335	10871.1735	9979.6839	9072.0375	8151.8536	7222.7076	6288.1477	5351.7205	4417.0075	3487.6759
5 AD	8.8329	8.9693	9.0985	9.2176	9.3241	9.4149	9.4873	9.5382	9.5626	9.5644	9.5289	9.4591	9.3484
5 NA	3.3165	3.3653	3.4162	3.4688	3.5228	3.5780	3.6340	3.6904	3.7467	3.8023	3.8563	3.9078	3.9555
5 α^2	-6.713	-6.789	-6.859	-6.923	-6.978	-7.023	-7.057	-7.078	-7.085	-7.075	-7.048	-7.000	-6.930
5 α^3	-0.694	-0.638	-0.581	-0.524	-0.467	-0.409	-0.352	-0.295	-0.240	-0.185	-0.132	-0.081	-0.032
5 α^4	-0.006	-0.006	-0.005	-0.005	-0.004	-0.004	-0.004	-0.003	-0.003	-0.002	-0.002	-0.002	-0.001
5 Total	14212.6878	13423.2833	12602.7041	11754.7747	10883.2755	9991.9332	9084.4176	8164.3475	7235.2859	6300.7862	5364.3875	4429.6661	3500.2837
5 δ	.0028	.0028	.0029	.0029	.0029	.0030	.0030	.0031	.0031	.0032	.0032	.0032	.0032

TABLE V: Components of dissociation energy of HD

$v \setminus J$	13	14	15	16	17	18	19	20	21	22	23	24	25
6 BO	11752.5477	11010.2654	10239.7570	9444.8355	8629.2939	7796.9006	6951.4042	6096.5448	5236.0757	4373.7974	3513.6067	2659.5698	1816.0315
6 AD	8.7046	8.7900	8.8644	8.9250	8.9689	8.9931	8.9944	8.9694	8.9146	8.8257	8.6981	8.5261	8.3027
6 NA	3.6371	3.6764	3.7168	3.7579	3.7993	3.8405	3.8808	3.9196	3.9560	3.9887	4.0162	4.0365	4.0466
6 α^2	-6561	-6615	-6661	-6700	-6727	-6743	-6745	-6732	-6700	-6649	-6575	-6473	-6341
6 α^3	-0584	-0532	-0480	-0426	-0373	-0320	-0267	-0215	-0165	-0115	-0067	-0021	.0022
6 α^4	-0005	-0005	-0005	-0004	-0004	-0003	-0003	-0003	-0002	-0002	-0001	-0001	-0001
6 Total	11764.1743	11022.0166	10251.6236	9456.8054	8641.3516	7809.0275	6963.5779	6108.7389	5248.2595	4385.9352	3525.6567	2671.4828	1827.7489
6 δ	.0030	.0031	.0031	.0031	.0032	.0032	.0032	.0033	.0033	.0033	.0033	.0033	.0033
7 BO	9481.7095	8788.2487	8069.7908	7330.1868	6573.3025	5803.0243	5023.2747	4238.0424	3451.4272	2667.7112	1891.4663	1127.7257	382.2716
7 AD	8.3996	8.4314	8.4482	8.4472	8.4251	8.3788	8.3047	8.1988	8.0566	7.8727	7.6404	7.3509	6.9911
7 NA	3.8978	3.9253	3.9525	3.9787	4.0034	4.0257	4.0444	4.0582	4.0654	4.0636	4.0496	4.0187	3.9640
7 α^2	-6343	-6372	-6393	-6402	-6399	-6380	-6344	-6289	-6210	-6105	-5968	-5793	-5569
7 α^3	-0483	-0435	-0386	-0337	-0288	-0239	-0191	-0144	-0098	-0054	-0012	.0028	.0065
7 α^4	-0005	-0004	-0004	-0004	-0003	-0003	-0002	-0002	-0002	-0001	-0001	-0001	.0000
7 Total	9493.3238	8799.9242	8081.5132	7341.9385	6585.0622	5814.7666	5034.9701	4249.6560	3462.9182	2679.0314	1902.5582	1138.5187	392.6763
7 δ	.0034	.0033	.0033	.0034	.0034	.0034	.0034	.0034	.0034	.0034	.0033	.0033	.0033
8 BO	7395.5661	6753.2298	6089.5337	5408.4448	4714.0009	4010.3337	3301.7091	2592.5943	1887.7620	1192.4601	512.7007		
8 AD	7.9156	7.8916	7.8485	7.7831	7.6921	7.5715	7.4173	7.2242	6.9858	6.6939	6.3359		
8 NA	4.0864	4.0982	4.1075	4.1133	4.1144	4.1094	4.0961	4.0720	4.0336	3.9756	3.8906		
8 α^2	-6051	-6053	-6042	-6018	-5977	-5917	-5834	-5724	-5583	-5404	-5176		
8 α^3	-0390	-0346	-0301	-0256	-0211	-0167	-0123	-0081	-0041	-0003	.0033		
8 α^4	-0004	-0004	-0003	-0003	-0002	-0002	-0002	-0001	-0001	-0001	.0000		
8 Total	7406.9236	6764.5794	6100.8550	5419.7136	4725.1884	4021.4061	3312.6267	2603.3098	1898.2188	1202.5889	522.4130		
8 δ	.0034	.0034	.0034	.0034	.0034	.0034	.0034	.0034	.0033	.0033	.0032		
9 BO	5503.8623	4916.0122	4311.0380	3693.1508	3066.7301	2436.3848	1807.0535	1184.1722	573.9630				
9 AD	7.2548	7.1735	7.0686	6.9364	6.7731	6.5736	6.3321	6.0404	5.6867				
9 NA	4.1834	4.1731	4.1569	4.1330	4.0992	4.0526	3.9894	3.9041	3.7882				
9 α^2	-5672	-5641	-5594	-5528	-5440	-5326	-5181	-4997	-4764				
9 α^3	-0305	-0264	-0223	-0183	-0142	-0103	-0065	-0029	.0006				
9 α^4	-0003	-0003	-0003	-0002	-0002	-0002	-0001	-0001	-0001				
9 Total	5514.7025	4926.7680	4321.6815	3703.6489	3077.0439	2446.4679	1816.8504	1193.6140	582.9620				
9 δ	.0035	.0034	.0034	.0034	.0034	.0034	.0033	.0032	.0031				
10 BO	3820.2124	3291.7133	2751.2459	2203.5006	1653.5341	1106.9342	570.1135	50.8785					
10 AD	6.4252	6.2852	6.1162	5.9138	5.6721	5.3833	5.0359	4.6103					
10 NA	4.1591	4.1166	4.0624	3.9934	3.9053	3.7920	3.6444	3.4472					
10 α^2	-5190	-5118	-5025	-4906	-4755	-4566	-4328	-4022					
10 α^3	-0228	-0191	-0154	-0118	-0083	-0049	-0017	.0000					
10 α^4	-0003	-0002	-0002	-0002	-0001	-0001	-0001	.0000					
10 Total	3830.2546	3301.5839	2760.9064	2212.9053	1662.6275	1115.6479	578.3592	58.5350					
10 δ	.0034	.0034	.0033	.0033	.0032	.0031	.0030	.0028					
11 BO	2363.5820	1901.6070	1434.3652	967.5491	507.7385	63.0112							
11 AD	5.4384	5.2367	4.9980	4.7147	4.3751	3.9588							
11 NA	3.9694	3.8781	3.7649	3.6233	3.4431	3.2070							
11 α^2	-4580	-4455	-4299	-4104	-3859	-3545							
11 α^3	-0158	-0126	-0094	-0063	-0033	-0005							
11 α^4	-0002	-0002	-0001	-0001	-0001	-0001							
11 Total	2372.5158	1910.2635	1442.6887	975.4703	515.1674	69.8220							
11 δ	.0033	.0032	.0031	.0030	.0028	.0026							

TABLE VI: Components of dissociation energy of HD

$v \setminus J$	13	14	15	16
12 BO	1160.9721	776.7464	397.0753	30.3683
12 AD	4.3011	4.0256	3.6946	3.2853
12 NA	3.5462	3.3760	3.1621	2.8840
12 α^2	-.3800	-.3599	-.3346	-.3019
12 α^3	-.0098	-.0070	-.0043	-.0017
12 α^4	-.0001	-.0001	-.0001	-.0001
12 Total	1168.4295	783.7811	403.5930	36.2340
12 δ	.0029	.0028	.0026	.0024
13 BO	253.6399			
13 AD	2.9805			
13 NA	2.7712			
13 α^2	-.2772			
13 α^3	-.0045			
13 α^4	-.0001			
13 Total	259.1098			
13 δ	.0023			

TABLE VIII: Components of dissociation energy of HD

$v \setminus J$	26	27
6 BO	987.7868	180.3678
6 AD	8.0184	7.6597
6 NA	4.0424	4.0175
6 α^2	-.6170	-.5950
6 α^3	.0063	.0101
6 α^4	.0000	.0000
6 Total	999.2369	191.4601
6 δ	.0033	.0033