Forecasting the Volatility of Volatility with Parametric Models

Introduction

During the last twenty years there is a dynamic development of the theory and practice of risk management. The important notions in researches is the market risk joined with the changes in prices of financial instruments. Evidently the basic characteristic of risk is a volatility. An important measure of volatility is the volatility implied from options prices, which may be interpreted as the volatility forecasted by the participants of the market (Płuciennik, Buszkowska, 2006).

In the study the author focuses on volatility of VIX (the VVIX Index). The basic VIX Index is a popular measure of the implied volatility from S&P 500 Index options. VVIX is a an index of implied volatility of VIX from the Chicago Board Options Exchange Market Volatility Index and it is accessible on the website (www.cboe.com). VVIX Index is calculated on the basis of VIX. It is often referred to as the fear index. It represents one measure of the market's expectation of stock market volatility over the next 30 day period. VVIX In-

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dex is an indicator of the expected volatility of the 30-day forward price of VIX.

The intention of the author is to analyze if ARFIMA model is an appropriate model to predict VVIX Index in the period of crisis. To meet stationarity and forecasting properties of the series modifications of VVIX are applied such as logarithms of the series, pre-differenced series and first differences of logarithms. Log transformation may improve forecasts of economic variables in some cases. This is true when "log transformation stabilizes the variance of the underlying series" (Lutkepohl, H. 2009). Another purpose is to check the econometric properties of VVIX like stationarity, long memory and heteroskedasticity in the periods of crisis. The author compares this empirical observations with the results for implied volatility from the paper of Płuciennik and Buszkowska, (2006). What is more the author creates conditional volatility of volatility and analyze if it can be applied to predict future behavior of the market. The importance of VVIX stems from the possibility of predicting stock market crashes / downturns / crises. The results of **Jared Woodard in the paper Fear of unknown, volatility of volatility predicts stock returns** reveal that, compared to otherwise similar stocks in the sample from 1996 to 2009, stocks with a higher volatility-of-volatility (vol of vol) had significantly lower future returns. The volatility of volatility is an important indicator of a future behavior of the market. This dependence is illustrated on the figure 1, the figure 2, the figure 3 and the figure 4.

The author hypothesizes that ARFIMA model in subprime crisis was a good predictor of VVIX Index, because it may be suitable to model and forecast implied volatility (Płuciennik and Buszkowska 2006). What is more in time of crisis VVIX Index series have short memory dependence, are not stationary, as it is true for the implied volatility in general. Another hypothesis says that conditional volatility of volatility in contrast to VVIX Index is not a good indicator of future growths of S&P 500, because generally the behavior of conditional variance is different than that of implied volatility.

Forecasting volatility of volatility with ARFIMA, knowing the statistical properties of VVIX and the comparison with implied volatility in crisis delivers more information about the last financial crisis. Constructing conditional volatility of volatility, which is not so known as VVIX in terms of applications, is a permanent result of the survey.

The author makes use of GARCH family of models, the classical GARCH(1,1) by Bollerslev (1986) and the EGARCH model by Nelson (1991). Following Płuciennik and Buszkowska (2006) she decides to forecast VVIX series with ARFIMA and constructs conditional volatility of volatility applying GARCH(1,1) to VIX index. The forecasts are compared by the mean of Mean Squared Error (MSE) and also by Root Mean Squared Deviation (RMSE). The author also researches natural logarithm of VVIX, the first differences of VVIX, and first differences of logarithms of VVIX.

If time series exhibits long memory dependence its dynamics possibly may be described with ARFIMA models [Granger, Joyeux, 1980], Hosking (1981). Let $\phi(L)$ and $\theta(L)$ denote lag polynomials of the form

$$\eta(L) = 1 - \eta_1 L - \ldots - \eta_r L^r \qquad (1)$$

of the order *p* and *q* respectively. The ARFIMA(p,d,q) model is given by the formula:

$$\phi(L)(1-L)^d (y_t - \gamma) = \theta(L)\varepsilon_t, \qquad (2)$$

where ε_t is a sequence of independent and identically distributed random variables an *d* is the fractional integration parameter. The expression $(1-L)^d$ is called the fractional difference operator and equals

$$(1-L)^d = \sum_{i=0}^{\infty} b_i L^i, \qquad (3)$$

where

$$b_0 = 1 \text{ and } b_i = \frac{-d\Gamma(i-d)}{\Gamma(1-d)\Gamma(i+1)} = \frac{i-d-1}{i} b_{i-1} \text{ for } i \geq 1$$

In practice the infinity in the formula is replaced by *t-1*.

There are different methods of detecting the long memory dependence. A semi-parametric approach to test long-memory was proposed by Geweke and Porter–Hudak (1983). Let's take the fractionally integrated process x_t. The spectra density of the proces expresses the following formula:

$$f(\omega) = [2\sin(w/2)]^{-2d} f_u(\omega), \tag{4}$$

where ω is the Fourier frequency, $f_u(\omega)$ is the spectra density corresponding to u_t and u_t is a stationary short memory disturbance with expected value equal zero. Calculating the natural logarithm of each side and simplifying they obtain:

$$\ln f(\omega_j) = \ln f_u(\omega_j) - d\ln[4\sin^2(\omega_j/2)] \tag{5}$$

where ω_j is the set of harmonic frequencies ($\omega_j = 2\pi j/n$) for $j = 0,1,\ldots,n/2$, and n is the sample size. Geweke and Porter – Hudak (1983) shows that using an estimate of $f(\omega)$, if the number of frequencies in the regression (5) is a positive integer function $g(n)=\left[n^{\alpha}\right]$ ($0<\alpha<1$,we can estimate $\hat{d}$ by least squares method on the basis of (5). The OLS estimator is asymptotically normally distributed in large samples:

$$\hat{d} \sim N\left(d, \frac{\pi^2}{6\sum_{j=1}^{g(n)}\left(U_j-\overline{U}\right)^2}\right) \tag{6}$$

where
$U_j=\ln\left[4\sin^2\left(\omega_j/2\right)\right]$, and $\overline{U}$ is the estimator of mean of U_j for $j=0,1,\ldots,\ g(n)$. The null hypothesis that the time series has no long memory (d=0) may be tested. If it is true the *t*- statistic:

$$t_{d=0}=\hat{d}\cdot\left(\frac{\pi^2}{6\sum_{j=1}^{g(n)}\left(U_j-\overline{U}\right)^2}\right) \tag{7}$$

has in limit a standard normal distribution.

1. Data

The author takes into account the VVIX Index. The frames of crisis in the research are indicated on the base of the daily Libor - OIS spreads which are the indicator of a fear of insolvency, (Sengupta , Yu, 2008), (Thornton , 2009). The crisis period is posited from 8 August 2007 to

9 March 2009 – see Blackburn (2008), Tudor (2009), Buszkowska and Płuciennik (2013)

The null hypothesis of stationarity is rejected for the VVIX Index and the logarithm of VVIX Index, but for the process of the first difference the author can't reject the H_0 hypothesis. The results are presented in Table1. The descriptive statistics of the VVIX series and the logarithm of volatility of volatility series are presented in Table 2.

Table 1. **The results of KPSS test**

Data	KPSS
VVIX	Test statistic: 0.515892 10% 5% 1% Critical value: 0.120 0.148 0.217
ln(VVIX)	Test statistic: 0.970977 10% 5% 1% Critical value: 0.120 0.148 0.217
first differences of VVIX	Test statistic: 0.0445818 10% 5% 1% Critical value: 0.120 0.148 0.217
first differences of ln of VVIX	Test statistic: 0.0480253 10% 5% 1% Critical value: 0.120 0.148 0.217

Source : own computations.

Table 2. **The descriptive statistics of the VVIX, the logarithm VVIX series an, the first differences process of VVIX and its log transformation**

Data	Mean	Std. dev.	Max.	Min.	Skew	Kurto-sis
VVIX	86.6584	15.3549	142.99	59.74	0.85	3.41137
ln(VVIX)	4.44722	0.169836	4.96277	4.09	0.46	2.70423
first differences of VVIX	0.0103509	4.05135	21.32	-9.96	1.04	6.29411
first differences of ln (VVIX)	6.63441 e-005	0.0448	0.2238	0.12	0.94	5.26923

Source : own computations.

The author rejects the null hypothesis of the GPH test for VVIX and the natural logarithm of VVIX. She can't reject the null hypothesis for the first differences of logarithm of VVIX. The lag (m =20) emerges from the length of the sample.

Table 3. **The results of the long memory depependence test for lag=20**

Data	GPH	p-value
VVIX	5.43991	0.0000
ln(VVIX)	5.57818	0.0000
first differences of VVIX	-0.681315	0.5043
first differences of ln (VVIX)	-0.319853	0.7528

Source : own computations.

2. Empirical Research about Variance

At first the author tests the conditional heteroskedasticity of the series using the Engle's test. The results are presented in table 4.

Table 4. **The results of conditional hereroskedasticity test**

Series	Engle	p-value
VVIX	5.8	0.016
logarithm of VVIX	349.858	0
first differences of VVIX	13.7385	0
first differences of logarithm of VVIX	1.6257	0.202

Source : own computations.

The Engle's test rejects conditional homoscedasticity for the first three series very strongly. Therefore, the author assumes that this series have a varying conditional variance. Hence, she models and forecasts them with GARCH models. The models ARFIMA(1,d,1)-EGARCH(1,1) and ARFIMA(0,d,0)-FIEGARCH(1,1) are well fitted to the data. ARFIMA(0,d,0) is an autoregressive fractionally integrated moving average as in Davidson (2006).

Table 5. **The parameters of ARFIMA (1,d,1)-EGARCH(1,1) model for VVIX and ARFIMA (0,d,0)-FIEGARCH(1,1) for logarithm of VVIX**

logarithm of VVIX	**VVIX**			Parameter	**logarithm of VVIX**		
	Estitimate	Std. Err	p-value		Estimamate	Std. Err.	p-value
Student's d.f.	3.91015	0.8845	NA	Student's t d.f.	8.11695	3.4181	NA
Intercept	59.5916	14.0578	0	Skewness	1.31794	0.0892	
				ARFIMA d	0.96954	0.01165	0
				ω	1.42998	0.2833	
ARFIMA d	0.64794	0.07119	0	FIEGARCH d	0.98536	0.2839	
AR1	0.8503	0.05519	0	EGARCH α_1	-0.75144	0.2682	0.005
MA1	0.74027	0.04966	0				
EGARCH Intercept	0.1679	0.3095	0	EGARCH β_1	0.7992	0.0403	0
EGARCH α_1	0.21751	0.10501	0.039				
EGARH β_1	0.88278	0.12513	0				

Source : own computations

The parameter in ARFIMA model $|d| > 0.5$ suggests that the original series is not stationary. It indicates the long memory in the process. The sum of the parameters of the of GARCH model is below one, so GARCH is stationary and all the parameters are significant. The AR parameter is below one as well so the model is stationary.

Tabele 6. **The estimated parameters of ARMA(0,1) – EGARCH models for VVIX and ARMA(0,1) for the logarithm of VVIX. Estimation on pre-differenced series.**

Parameter	VVIX			Parameter	Logarithm of VVIX		
	Estimate	Std. Err	p-value		Estimate	Std. Err.	---
Student's t d.f.	4.05202	1.0065	------	Student's t d.f.	6.36584	1.9345	---
MA1	0.17578	0.05444	0.001	Skewness ξ	1.36207	0.0921	---
EGARCH ω	0.07963	0.1898	------	MA1	0.18131	0.0501	0
EGARCH α_1	0.29104	0.05737	0	Error Variance	0.00195	0.0002	---
EGARCH β_1	0.89743	0.07064	0				---

Source : own computations

Both null hypotheses d=0 are not rejected. It means that the both time series are stationary and have short memories. That is why the author also inspects differences of the series.

One-day ahead volatility forecasts of the four series deliver forecast errors presented in Table 7. The author concludes that the errors are much smaller for the logarithms of the volatility of volatility and they are similar for the pre-differenced series. This conclusion is consistent with the results for classical implied volatility of WIG20 Index [Płuciennik, Buszkowska, 2006].

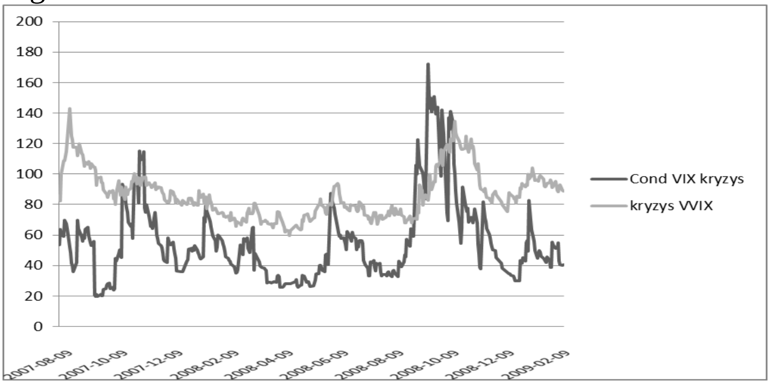
Table 7. **The forecast errors for 50 ex-post forecasts**

	Relative MSE	MSE	Relative RMSE	RMSE
VVIX	0,00093	7,87069	0,030552	2,805475
ln(VVIX)	4,55922E-05	0,000928	0,006752	0,030469
first differences of VVIX	1,337015	7,966892	1,156294	2,822568
VVIX	1,34755	0,00097	1,160829	0,031102

Source : own computations.

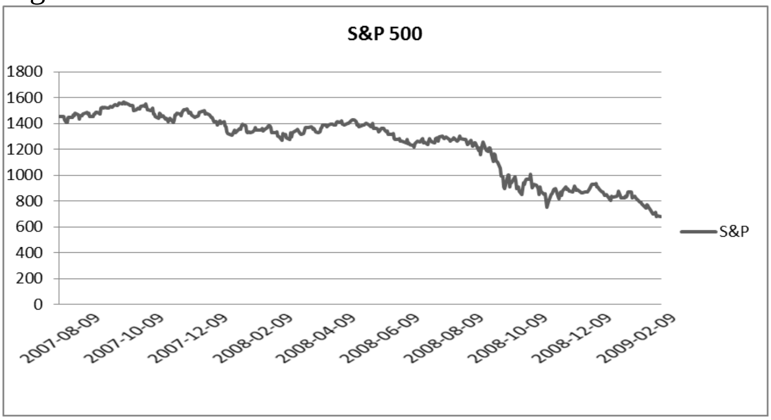
Next the author illustrates the plots of VVIX and the conditional volatility of VIX, so the volatility of volatility from GARCH(1,1). in the period of *subprime* crisis the both series produce high realizations. High volatility in both cases occurs before the falls on the stock market of S&P500 – see Figure 1. In the period of debt crisis VVIX is calm but the conditional volatility of VIX is high – see Figure 4 The calm VVIX occurs before the growths of S&P 500 and the high volatility is observed before the falls of this American index. It confirms the thesis of Woodard. This is not true for conditional volatility of VIX.

Figure 1. **VVIX and Conditional VVIX in crisis**



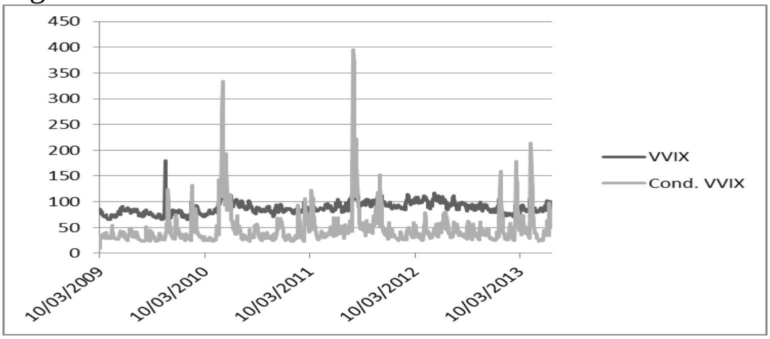
Source: own.

Figure 2. **S&P 500 in crisis**



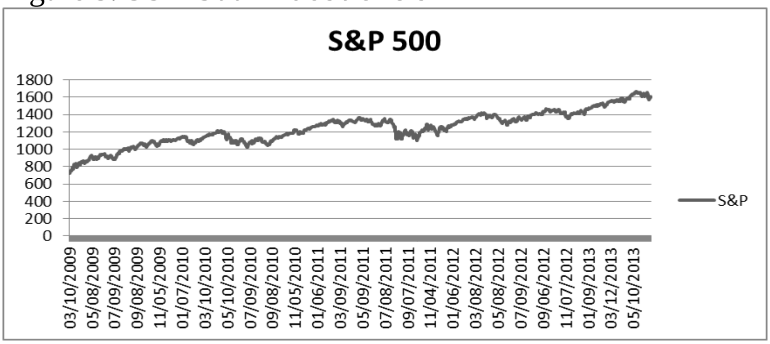
Source: own.

Figure 4. **VVIX and Conditional VVIX in debt crisis**



Source: own.

Figure 3. **S&P 500 in debt crisis**



Source: own.

With Time Series Modelling 4.20 of Davidson (2006) the author obtains estimates of GARCH(1,1) model for conditional volatility of VIX for the first period (figure 1). Box-Pierce tests of autocorrelations in standardized residuals and squared residuals do not indicate significant correlations

Table 8. **The estimates of parameters for forecasting GARCH(1,1) model for VIX in crisis**

Parameters	Estimate	Std. Err.	p-Value
Student's t d.f.	4.89515	1.1168	------
Skewness ξ (*of skew Student's t-distribution*)	1.17489	0.0803	------
GARCH ω	6.63431	4.1879	------
GARCH α_1	0.10106	0.03462	0.004
GARCH β_1	0.80631	0.07438	0

Source: own.

The model of conditional volatility of VIX for the period after crisis presented on the figure 3 The tests of autocorrelations in standardized residuals and squared residuals indicate good fitting. All the parameters are significant.

Table 9. **The estimates of parameters for forecasting ARMA - GARCH(1,1) model for VIX after crisis**

Parameters	Estimate	Std. Err.	p-Value
GED nu	1.26316	0.0814	------
AR1	0.79868	0.05417	0
MA1	0.87812	0.04763	0
GARCH ω	7.03274	2.534	------
GARCH α_1	0.1437	0.03738	0
GARCH β_1	0.70345	0.7944	0

Source: own.

3. Conclusions

The author concludes that it is impossible to predict future behavior of the market based on conditional volatility of volatility unlike VVIX which may be used to forecast the growths and falls of S&P 500. It may emerge from the fact that the behavior of conditional variance and implied volatility is different in general. The econometric characteristic of volatility of volatility in periods of crisis is very similar to that of classical implied volatility of the WIG20 Index analyzed in the paper (Płuciennik, Buszkowska, 2006).

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Abstrakt

The purpose of the research is the assessment of forecasts performance of the VVIX Index, in other words the volatility of volatility index. The survey concerns the periods including the subprime crisis and the debt crisis. The intention of the author is to get to know the forecasting properties of volatility of volatility and to compare its econometric characteristics with that of classical implied volatility in subprime crisis. The significance of volatility of volatility emerges from the possibility of predicting the falls of exchange quotations on a stock market on the basis of it.

Keywords: VVIX index, VIX index, ARFIMA model, GARCH model, implied volatility.

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